Name ____
ID number______________________________________
Section A and B

Calculus II (Math 122) Final Exam, 12 May 2011

This is a closed book exam. No notes or calculators are allowed. A table of trigonometric identities is attached. To receive credit you must show your work. Make sure you read the question closely and answer each part. Please turn off all cell-phones and other electronic devices. When you are finished please write and sign the honor code, (I have neither given nor received unauthorized aid on this exam) in the space provided below. Please remember that you are also obligated to report violations of the honor code.
Each of the nine problems is worth 11 points, one additional point is given for your favorite mathematics joke.

1	
2	
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9	
10(joke)	
Total	

Honor Code:

Signature:

1. Compute the following limit:

$$\lim_{n\to\infty} (1 + 1/n)^n$$

2. Find a power series representation for the function $f(x) = \frac{1+x}{1-x}$ and determine the interval of convergence. (Note that there are several easier methods than Taylor's method to obtain the representation.)

3. Find the Maclaurin series for the function $g(x) = \ln(1 + x)$ using the definition of a Maclaurin series. Then determine the radius of convergence for the series obtained.

4. Given that the function e^x has the power series representation $\sum_{n=0}^{\infty} \frac{x^n}{n!}$, give an approximation to the following definite integral and give an upper bound on your error.

$$\int_0^{0.5} x^2 e^{-x^2}\, dx$$

5. Use Taylor's Inequality to show that the power series representation for e^x is indeed $\sum_{n=0}^{\infty} \frac{x^n}{n!}$.

6. Use Euler's method with step size 0.5 to estimate $y(1)$, where $y(x)$ is the solution of the initial-value problem $y' = 1 - xy, \quad y(0) = 0$.

7. Solve the following separable differential equation.

$$\frac{dy}{d\theta} = \frac{e^y \sin^2 \theta}{y \sec \theta}$$

8. Find the solution of the initial-value problem

$$x^2y' + xy = 1 \quad x > 0 \quad y(1) = 2$$

9. Find the sum of the infinite series

$$1+\frac{2}{10^1}+\frac{3}{10^2}+\frac{7}{10^3}+\frac{2}{10^4}+\frac{3}{10^5}+\frac{7}{10^6}+\frac{2}{10^7}+\frac{3}{10^8}+\frac{7}{10^9}+\dots$$

10. Write your favorite mathematics joke here.

Trigonometric Identities

Addition and subtraction formulas

- $\sin(x+y) = \sin x \cos y + \cos x \sin y$
- $\sin(x-y) = \sin x \cos y - \cos x \sin y$
- $\cos(x+y) = \cos x \cos y - \sin x \sin y$
- $\cos(x-y) = \cos x \cos y + \sin x \sin y$
- $\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$
- $\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

Double-angle formulas

- $\sin(2x) = 2 \sin x \cos x$
- $\cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$
- $\tan(2x) = \frac{2 \tan x}{1 - \tan^2 x}$

Half-angle formulas

- $\sin^2 x = \frac{1 - \cos(2x)}{2}$
- $\cos^2 x = \frac{1 + \cos(2x)}{2}$

Others

- $\sin A \cos B = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$
- $\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$
- $\cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$