Name ______________________________________
ID number____________________________________
Section C

Calculus II (Math 122) Final Exam, 13 May 2008

This is a closed book exam. No notes or calculators are allowed. A table of trigonometric identities is attached. To receive credit you must show your work. Please turn off all cell-phones and other electronic devices. When you are finished please write and sign the honor code, (I have neither given nor received unauthorized aid on this exam) in the space provided below. Please remember that you are also obligated to report violations of the honor code.

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Honor Code:

Signature:

1. (4 points each = 2 for correct answer + 2 for justification) Mark each of the following true (T) or false (F). If the statement is false, indicate why it is so. If the statement is true, justify your claim.

   (a) ______ If $\{a_n\}$ and $\{b_n\}$ are divergent, then $\{a_n + b_n\}$ is divergent.

   (b) ______ If $f(x) \leq g(x)$ and $\int_0^\infty g(x)dx$ diverges, then $\int_0^\infty f(x)$ also diverges.

   (c) ______ The Taylor series of $y = e^x$ centered at 1 is $\displaystyle\sum_{n=0}^{\infty} \frac{e^1 \cdot (x)^n}{n!}$.

   (d) ______ If $\lim_{n\to\infty} a_n = 0$, then $\displaystyle\sum_{n=1}^{\infty} a_n$ is convergent.

   (e) ______ The series $\displaystyle\sum_{n=1}^{\infty} n^{-\pi+1}$ is convergent.

   (f) ______ $0.9999999\ldots = 1$.

2. (8 points) Determine the convergence set (the interval of convergence) for $\sum_{n=1}^{\infty}(-1)^n \frac{x^n}{n^2 5^n}$.

3. (8 points) Find the area inside $r^2 = 4\cos(2\theta)$. Please include a sketch of the curve (on the sheet which follows).

4. (8 points) Find the exact length of the curve:

$$x = 1 + 3t^2, \quad y = 4 + 2t^3, \quad 0 \leq t \leq 1$$

5. (8 points each) Determine the following.

(a) $\int 6x^2 \cos(x^3)dx$

(b) $\int t^2 \cos(t)dt$

(c) This problem has no elementary solution. Express your answer as a series.

$$\int \cos(t^4)dt$$

6. (8 points each) Solve the differential equations and initial value problems

(a) $\dfrac{dy}{dx} = 2y(1-y)$.

(b) $y' = 5 + y, \qquad y(0) = 2$. (Hint: this is a linear differential equation.)

7. (12 points) A small pond has been contaminated with 100 gallons of human fecal matter. In total the pond holds 200,000 gallons of liquid. Uncontaminated water from a nearby stream is diverted into the pond at the rate of 1,000 gallons an hour and water is pumped from the pond into a fleet of trucks at the same rate. If the trucks can only hold 10,000 gallons, determine the amount of fecal matter in the pond when the trucks are full. (Assume that the pond is well-mixed, ugh.)

Trigonometric Identities

Addition and subtraction formulas

- $\sin(x + y) = \sin x \cos y + \cos x \sin y$
- $\sin(x - y) = \sin x \cos y - \cos x \sin y$
- $\cos(x + y) = \cos x \cos y - \sin x \sin y$
- $\cos(x - y) = \cos x \cos y + \sin x \sin y$
- $\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$
- $\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

Double-angle formulas

- $\sin(2x) = 2 \sin x \cos x$
- $\cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$
- $\tan(2x) = \frac{2 \tan x}{1 - \tan^2 x}$

Half-angle formulas

- $\sin^2 x = \frac{1 - \cos(2x)}{2}$
- $\cos^2 x = \frac{1 + \cos(2x)}{2}$

Others

- $\sin A \cos B = \frac{1}{2}[\sin(A - B) + \sin(A + B)]$
- $\sin A \sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$
- $\cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$