Name ____
ID number______________________________________
Sections **C**

Calculus II (MATH 0122) Final Exam, 14 December 2016

This is a closed book exam. Notes, calculators, cell-phones are not allowed – the only allowable items are pens, pencils and erasers. A table of trigonometric identities is attached. To receive credit you must show your work. Please leave answers as square roots, $\ln(), \exp()$, fractions, or in terms of constants like e, π, etc. Please turn off all cell-phones and other electronic devices. When you are finished please write and sign the Honor Code (I have neither given nor received unauthorized aid on this exam. I have not witnessed another giving or receiving unauthorized aid.) in the space provided below. Good luck!

Please complete seven of the eight questions. Indicate which you will omit by putting a slash through that question.

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Total	

Honor Code:

Signature:

1. [10 points] Find the radius of convergence **and** interval of convergence of the series.

$$\sum_{n=1}^{\infty} \frac{(-1)^n 4^n}{\sqrt{n}} x^n$$

2. [10 points] Find a power series representation for the function by first using partial fractions **and** determine the interval of convergence.

$$f(x) = \frac{2x+3}{x^2+3x+2}$$

3. [10 points] Given that $\ln(1+x) = \sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$, evaluate the indefinite integral as a power series, what is the radius of convergence? (Caution: please note that the argument for the natural logarithm function given above is different to the argument when this function appears as part of the integrand below.)

$$\int x^2 \ln(1+x^2)dx$$

4. [10 points] Find the Taylor series for $f(x) = \sin(x)$ centered at the value of π. [Assume that $f(x)$ has a power series expansion. Do not show that $R_n(x) \to 0$.] Also find the radius of convergence.

5. [10 points] [R. Nelsen] Consider the figure below. Use the figure to explain the following implication:

$$x \in [0, 1) \Rightarrow 1 + 2x + 3x^3 + 4x^3 + \cdots = (\frac{1}{1-x})^2$$

6. [10 points] Consider the following differential equation

$$\frac{dv}{dt} = -v[v^2 - (1+a)v + a],$$

where a is a positive constant such that $0 < a < 1$.

(a) For what values of v is $\frac{dv}{dt} = 0$?

(b) For what values of v is v increasing?

(c) For what values of v is v decreasing?

7. [10 points] Use Euler's method with step size 0.5 to estimate $y(1)$, where $y(x)$ is the solution of the initial-value problem $y' = x^2y - \frac{1}{2}y^2, \quad y(0) = 1$. (That is, find (x_2, y_2).)

8. [10 points] Solve the following separable differential equation.

$$\frac{dy}{dx} = \frac{x\sin(x)}{y}, \quad y(0) = -1$$

Trigonometric Identities

Addition and subtraction formulas

- $\sin(x+y) = \sin x \cos y + \cos x \sin y$
- $\sin(x-y) = \sin x \cos y - \cos x \sin y$
- $\cos(x+y) = \cos x \cos y - \sin x \sin y$
- $\cos(x-y) = \cos x \cos y + \sin x \sin y$
- $\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$
- $\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

Double-angle formulas

- $\sin(2x) = 2 \sin x \cos x$
- $\cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$
- $\tan(2x) = \frac{2 \tan x}{1 - \tan^2 x}$

Half-angle formulas

- $\sin^2 x = \frac{1 - \cos(2x)}{2}$
- $\cos^2 x = \frac{1 + \cos(2x)}{2}$

Others

- $\sin A \cos B = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$
- $\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$
- $\cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$