Name _____
ID number_____
Please circle your section: A: 8:00 - 8:50, B: 9:05 - 9:55, C 10:10 - 11 or D 11:15 - 12:05

Calculus II (Math 122) Final Exam, 11 December 2007

This is a closed book exam. No notes or calculators are allowed. To receive credit you must show your work. When you are finished please write and sign the honor code, (I have neither given nor received unauthorized aid on this exam) in the space provided below.

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Honor Code:

Signature:

1. (2 points each) Mark each of the following true (T) or false (F).

   (a) _____ $\displaystyle\int_1^{3+\sqrt{2}} \frac{1}{t}\,dt = \ln(3) + \ln\sqrt{2}$.

   (b) _____ If for the sequence $\{a_k\}$ we have $a_k > a_{k+1} > 0$ for all integers $k > 0$ then $\lim_{k\to\infty} a_k$ exists.

   (c) _____ If $a_k > a_{k+1} > 0$ for all integers $k > 0$ then the series $\displaystyle\sum_{k=1}^{\infty} a_k$ converges.

   (d) _____ When the series $\displaystyle\sum_{k=0}^{\infty} (10x)^k$ converges, its sum is $\dfrac{1}{1-10x}$.

   (e) _____ If the series $\displaystyle\sum_{k=0}^{\infty} a_k$ diverges and the series $\displaystyle\sum_{k=0}^{\infty} b_k$ diverges, then the series $\displaystyle\sum_{k=0}^{\infty} (a_k - b_k)$ must diverge.

2. (8 points) Determine the convergence set (the interval of convergence) for $\displaystyle\sum_{k=1}^{\infty} \frac{(2x-3)^k}{k+16}$.

3. (8 points each) Determine if the following series converge or diverge. For alternating series that converge determine if the convergence is absolute or conditional. You must state or clearly demonstrate what test you are using to determine convergence. You will not receive full credit for just heuristic reasoning.

(a) $\sum_{k=1}^{\infty} ke^{-k^2}$.

(b) $\sum_{k=4}^{\infty} \frac{2-k^2}{(k-3)^2}$.

(c) $\sum_{k=1}^{\infty} (-1)^{k-1} \frac{k^2+3k}{k^3+4}$.

4. (8 points) Find the area inside $r = 3 + 2\sin(\theta)$ and outside $r = 4$.

5. (12 points) The folium of Descartes is defined by the parametric equations

$$x = \frac{3t}{1+t^3}, \quad y = \frac{3t^2}{1+t^3}.$$

(a) Find $\frac{dy}{dx}$.

(b) Find the points on the curve where the tangent lines are horizontal.

6. (8 points each) Determine the following.

(a) $\int \sqrt{9-x^2}\,dx.$

(b) $\int_0^4 \frac{1}{(x-3)^2}\,dx.$

(c) $\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{5x}.$

7. (8 points each) Solve the differential equations and initial value problems

(a) $\dfrac{dy}{dx} = y(y-5)$; $\qquad y(0) = 1.$

(b) $y' = y + 2xe^{2x}$; $\qquad y(0) = 3.$

8. A tank with a capacity of 500 gallons originally contains 200 gallons of water in which is dissolved 100 pounds of salt. Brine containing 1 pound of salt per gallon enters the tank at at rate of 3 gal/min. The well stirred mixture is allowed to flow out at 2 gal/min.

   (a) (8 points) Find the equation that will enable you to compute the amount of salt in the tank at any time prior to the instant the tank is about to overflow.

   (b) (6 points) Find the concentration (in pounds per gallon) of salt in the tank when the tank is totally full (that is, at the instant it is about to overflow).

(c) (6 points) Compare this concentration with the theoretical limiting concentration if the tank had infinite capacity.