Calculus II - Exam 3 - Spring 2012

April 26, 2012

Name:
Honor Code Statement:

Directions: Justify all answers/solutions. Calculators, notes and texts are not permitted. The point value of each problem is indicated in brackets. Good luck!

1. [4] Define what it means for a sequence to be **monotonic**.

2. [6] Prove that the sequence the terms of which are given by $a_n = \frac{2^n}{n!}$ converges.

3. [4] Find the sum of the following series.

$$2 + \frac{1}{2} + \frac{1}{8} + \frac{1}{32} \dots$$

4. [4] Theorem 6 of Section 11.2 states: If the series $\sum a_n$ is convergent, then $\lim_{n\to\infty} a_n = 0$. Give an example that shows that the converse of this statement is not true in general.

5. [6] Show that the following series diverges.

$$\sum_{n=1}^{\infty} \frac{e^n}{n^2}$$

6. [6] Use the Integral Test to show the following series converges. (Please check the hypothesis of the test.)

$$\sum_{n=1}^{\infty} \frac{1}{n^2+1}$$

7. [8 points each] Determine if the series converges or diverges. State the test that you use.

(a)

$$\sum_{n=1}^{\infty} \frac{1}{n!}$$

(b)

$$\sum_{n=1}^{\infty} (-1)^n \sin\left(\frac{\pi}{n}\right)$$

(c)

$$\sum_{n=1}^{\infty} \frac{\cos(\frac{n\pi}{2})}{n!}$$

8. [6] Write the number $2.3131313131\ldots$ as a ratio of integers.

9. [10] Find the radius of convergence *and* interval of convergence of the following series.

$$\sum_{n=1}^{\infty} \frac{(x+13)^n}{n}$$

10. [10] Without appealing to the method of Taylor and Maclaurin, find a power series representation for $f(x) = \ln(x^2 + 4)$. Give the interval of convergence. (Hint: begin by differentiating $f(x)$.)