Calculus II - Exam 3 - Spring 2011

April 21, 2011

Name:
Honor Code Statement:

Directions: Justify all answers/solutions. Make sure to indicate the test or theorem that you use. Calculators are not permitted. Good luck!

1. [10 points each] Determine if the following series are convergent or divergent.

   (a)

$$\sum_{n=1}^{\infty} \frac{n^{2n}}{(1+2n^2)^n}$$

(b)

$$\sum_{n=2}^{\infty} \frac{1}{n \ln n}$$

(c)

$$\sum_{n=1}^{\infty} \frac{2n}{2n^3 + 1}$$

2. [10 points] Determine if the following series is absolutely convergent, conditionally convergent or divergent.

$$\sum_{n=2}^{\infty} (-1)^n \frac{\sqrt{n}}{\ln n}$$

3. [5 points] If we use the fifth partial sum as an estimate for the following convergent series, give an error estimate and indicate whether the fifth partial sum is an over-estimate or under-estimate.

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2}{n^3 + 4}$$

4. [10 points] Find the sum of the following convergent series,

$$\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{2^{3n}}.$$

5. [5 points] If $\lim_{n\to\infty} a_n = 0$, then $\sum a_n$ is convergent. **True or false?** Justify your answer.

6. [5 points] $0.999\ldots = 1$. **True or false?** Justify your answer.

7. [5 points] If the sequence $\{a_n\}$ is decreasing and $a_n > 0$ for all n, then the sequence $\{a_n\}$ is convergent. **True or false?** Justify your answer.

8. [10 points] Show that the series $\sum_{n=1}^{\infty} \frac{n^n}{(2n)!}$ is convergent.

9. [10 points] Find the radius of convergence and interval of convergence of the series:

$$\sum_{n=1}^{\infty} \frac{2^n (x-2)^n}{(n+2)!}$$