

Calculus II - Exam 3 - Spring 2008

April 24, 2008

Name:

Honor Code Statement:

Directions: **Justify** all answers/solutions. Calculators are not permitted.

1. Determine if the following series is convergent or divergent. If it is convergent then give the sum.

- $60 + 30 + 15 + 7.5 + 3.75 + \dots$

- $0.005 + 0.01 + 0.02 + 0.04 + \dots$

- $\frac{1}{50} + \frac{1}{51} + \frac{1}{52} + \frac{1}{53} + \frac{1}{54} + \dots$

2. Determine if the following series are convergent or divergent.

(a)

$$\sum_{n=1}^{\infty} \frac{5 - 2\sqrt{n}}{n^3}$$

(b)

$$\sum_{n=1}^{\infty} \frac{1}{2n + 3}$$

(c)

$$\sum_{n=1}^{\infty} \frac{n^n}{n!}$$

(d)

$$\sum_{n=1}^{\infty} \frac{n^2 - 5n}{n^3 + n + 1}$$

(e)

$$\sum_{n=1}^{\infty} \left(-\frac{n}{5}\right)^n$$

3. Determine if the following series is absolutely convergent, conditionally convergent or divergent.

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^3 5^n}{n!}$$

4. Find the radius **and** interval of convergence of the following series.

$$\sum_{n=1}^{\infty} \frac{n}{4^n} (x+1)^n$$

5. Find a power series representation for the function $f(x) = \frac{x}{1+2x^2}$.

6. Is the following sequence bounded? Is it decreasing? (Justify.) What can you conclude about this sequence? $a_n = \frac{1}{2n+3}$

7. Find the Taylor series for $f(x) = \cos(x)$ centered at $a = \pi$.

8. Given that $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ for all real numbers. Give an estimate (which you may leave as a sum) for $\int_0^1 e^{-x^2} dx$ **AND** give an error estimate.