Calculus II - Exam 3 - Fall 2013

November 14, 2013

Name:
Honor Code Statement:

Directions: Upon completion of the examination and prior to its submission, please write and sign the Honor Code. **Justify** all answers/solutions. Make sure to indicate the test or theorem that you use. Calculators are not permitted, and all electronic devices should be off. Good luck!

1. [5 points] Give an example of a convergent geometric series with sum $\frac{8}{5}$. (The "tail" of the series should not consist of all zeros.)

2. [5 points] State the Monotonic Sequence Theorem and give an example of such a sequence that the theorem describes. [5 points]

3. [10 points] Use the Integral Test to determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} n^2 e^{-n^3}$$

4. [10 points] Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=1}^{\infty} (-1)^n \frac{n^5}{4^n}$$

5. [10 points] Find the radius of convergence and interval of convergence of the series.

$$\sum_{n=1}^{\infty} \frac{(2x-1)^n}{5^n \sqrt{n}}$$

6. [10 points] Find the Taylor series for $1/x$ at $a = -3$. Further, what is the largest open interval for which we can say that the Taylor series obtained *represents* $1/x$?

7. [5 points] Show how to use the Alternating Series Estimation Theorem to determine the number of terms one must take to estimate the sum of the Alternating Harmonic Series to within 0.005. (You needn't solve for n explicitly since you don't have a calculator and I don't want you spending time doing arithmetic.)

8. [5 points] The Riemann zeta-function ζ is defined by

$$\zeta(x) = \sum_{n=1}^{\infty} \frac{1}{n^x}$$

and is used in number theory to study the distribution of prime numbers. What is the domain of ζ?

9. [5 points] Suppose that $\sum_{n=1}^{\infty} a_n$ is a convergent series and no term of the series equals zero. Prove that $\sum_{n=1}^{\infty} 1/a_n$ is a divergent series.

10. [5 points] In the figure below, I have plotted $f(x) = x + e^{-x}$ and its third-degree Taylor polynomial $T_3(x)$. The Maclaurin Series for this function is $1 + \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{n+1}}{(n+1)!}$. Do the following three things: identify which curve is $T_3(x)$, compute $T_3(3)$ and use the figure to illustrate only the value of $R_3(3)$.