Calculus II - Exam 3 - Fall 2006

November 16, 2006

Name:
Honor Code Statement:

Directions: Justify all answers/solutions. Calculators are not permitted.

1. Define what it means for a sequence to **diverge to infinity.**

2. Define what it means for M to be a **least upper bound** for a sequence $\{a_n\}$.

3. Indicate which series converge and which diverge. **JUSTIFY** your answer. Indicate **which test** you use. You may **OMIT ONE** item from this question - indicate which you omit.

- $$\Sigma_{n=0}^{\infty} \frac{n!}{1,000,000^n}$$

- $$\Sigma_{n=1}^{\infty} \frac{1}{3^{n-1}+1}$$

  . (Give an upper bound for the sum.)

- $$\Sigma_{n=2}^{\infty} \frac{(\ln(n))^3}{n^3}$$

  (Hint: compare to $\Sigma_{n=2}^{\infty} \frac{1}{n^2}$.)

- $$\Sigma_{n=2}^{\infty} \frac{1}{n \ln(n)}$$

- $$\Sigma_{n=1}^{\infty}\frac{(n!)^n}{(n^n)^2}$$

- $$\Sigma_{n=1}^{\infty}\frac{n!}{n^n}$$

4. State and prove the Sandwich Theorem for Sequences.

5. Find the series' radius and interval of convergence. For what values of x does the series converge (a) absolutely? (b) conditionally?

$$\Sigma_{n=1}^{\infty}(\ln(n))x^n$$

6. Find the Taylor polynomial of order 4 generated by $f(x) = \cos(x)$ at $a = 0$.

This fourth order Taylor polynomial approximates $\cos(x)$. Use **Taylor's Formula** to estimate the error when $x = 1$.

7. Given that the power series for $\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} \ldots \frac{(-1)^n x^{2n+1}}{(2n+1)!} \ldots$. Estimate the following definite integral and give an error estimate.

$$\int_0^1 x^3 \sin(\sqrt{x})dx$$