Calculus II - Exam 2 - Spring 2011
Techniques of Integration

March 24, 2011

Name:
Honor Code Statement:

Directions: Justify all answers/solutions. Calculators are not permitted. You may use the table of trigonometric identities given on the last page. Each problem is worth 10 points. If you need extra space, use the blank white paper provided.

1. Evaluate each of the following integrals. If there is a particular technique that you use, name it.

   (a) $\int \cos^2 x \sin(2x)\ dx$

   (b) $\int \tan^3 x \sec x\ dx$

(c) $\int p^5 \ln(p)\ dp$

(d) Prove this identity: $\int x^n e^x\ dx = x^n e^x - n \int x^{n-1} e^x\ dx$.

(e) $\int \frac{x^2}{\sqrt{1-x^2}}\ dx$

(f) $\int_2^3 \frac{1}{x^2-1}\ dx$

2. Determine whether the following integrals converge or diverge. If the integral converges, determine the value to which it converges.

$$\int_2^3 \frac{1}{\sqrt{3-x}}\, dx$$

3. Use the (Direct) Comparison Theorem to determine whether the following integral is convergent or divergent.

$$\int_1^\infty \frac{1-e^{-x^2}}{x^4}\, dx$$

Trigonometric Identities

Addition and subtraction formulas

- $\sin(x+y) = \sin x \cos y + \cos x \sin y$
- $\sin(x-y) = \sin x \cos y - \cos x \sin y$
- $\cos(x+y) = \cos x \cos y - \sin x \sin y$
- $\cos(x-y) = \cos x \cos y + \sin x \sin y$
- $\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$
- $\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

Double-angle formulas

- $\sin(2x) = 2 \sin x \cos x$
- $\cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$
- $\tan(2x) = \frac{2 \tan x}{1 - \tan^2 x}$

Half-angle formulas

- $sin^2 x = \frac{1 - \cos(2x)}{2}$
- $\cos^2 x = \frac{1 + \cos(2x)}{2}$

Others

- $\sin A \cos B = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$
- $\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$
- $\cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$