Calculus II - Exam 2 - Spring 2008
Techniques of Integration

March 20, 2008

Name:
Honor Code Statement:

Directions: Justify all answers/solutions. Calculators are not permitted. You may use the table of trigonometric identities given on the last page.

1. Evaluate each of the following integrals.

   (a) $\int \tan^5 \theta \sec^3 \theta d\theta$

   (b) $\int_1^2 \frac{\sqrt{x^2-1}}{x} dx$. (Then use the answer to determine the average value of the function $\frac{\sqrt{x^2-1}}{x}$ on the interval $[1,2]$.)

(c) $\int \frac{dy}{y^2-4y-12}$

(d) $\int e^x \cos x dx$

(e) $\int \sin(5x)\cos(2x)dx$

2. Determine whether the following integrals converge or diverge. If the integral converges, determine the value to which it converges.

   (a) $\int_2^\infty \frac{dx}{x \ln x}$

   (b) $\int_0^1 \frac{dx}{2-3x}$

(c) $\int_1^\infty \frac{1+e^{-x}}{x} dx$. (Hint: use a comparison theorem.)

3. **True or False** Mark each of the following statements as either True or False. If the statement is false and can be easily modified to make it true then indicate how to do so or give an example which shows it is false.

   - If f is continuous on $[0, \infty)$ and $\int_1^\infty f(x)dx$ is convergent, then $\int_0^\infty f(x)dx$ is convergent.

   - If $f(x) \leq g(x)$ and $\int_0^\infty g(x)dx$ diverges, then $\int_0^\infty f(x)dx$ also diverges.

4. Evaluate the integral:

$$\int \frac{x^{1/3}+1}{x^{1/3}-1}dx$$

Trigonometric Identities

Addition and subtraction formulas

- $\sin(x + y) = \sin x \cos y + \cos x \sin y$
- $\sin(x - y) = \sin x \cos y - \cos x \sin y$
- $\cos(x + y) = \cos x \cos y - \sin x \sin y$
- $\cos(x - y) = \cos x \cos y + \sin x \sin y$
- $\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$
- $\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

Double-angle formulas

- $\sin(2x) = 2 \sin x \cos x$
- $\cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$
- $\tan(2x) = \frac{2 \tan x}{1 - \tan^2 x}$

Half-angle formulas

- $sin^2 x = \frac{1 - \cos(2x)}{2}$
- $\cos^2 x = \frac{1 + \cos(2x)}{2}$

Others

- $\sin A \cos B = \frac{1}{2}[\sin(A - B) + \sin(A + B)]$
- $\sin A \sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$
- $\cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$