# Calculus II - Exam 2 - Fall 2006

October 26, 2006

**Name:**
**Honor Code Statement:**

**Directions:** There are 13 problems. Do any **11** of these problems. **Justify** all answers/solutions. Calculators are not permitted. You may use the table of trigonometric identities given on the last page. Evaluate each given integral.

1. $\int \sin(y)e^{\cos(y)}dy$

$= -\int -\sin y \, e^{\cos(y)} dy = -\int e^u du$

let $u = \cos y$

then $du = -\sin y \, dy$

$= -e^u + C$

$= -e^{\cos y} + C$

2. $\int \frac{4x^3-x^2+16x \quad dx}{x^2+4}$

$= \int 4x - 1 + \frac{4}{x^2+4} dx$

$$\begin{array}{r} 4x \quad -1 \quad \\ x^2+4 \,\big)\, \overline{4x^3 - x^2 + 16x + 0} \\ -(4x^3 \quad\quad + 16x) \\ \hline -x^2 \quad + 0 \\ (-x^2 \quad\quad -4) \\ \hline 4 \end{array}$$

$= \frac{4x^2}{2} - x + 4\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$

1 $= 2x^2 - x + 2\tan^{-1}\left(\frac{x}{2}\right) + C$

3. $\int_0^{\pi/4} \sqrt{1 - \frac{2\tan(x)}{\tan(2x)}}\,dx$ (Hint: Use a trigonometric identity.)

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

$$1 - \tan^2 x = \frac{2\tan x}{\tan 2x}$$

$$1 - \frac{2\tan x}{\tan 2x} = \tan^2 x$$

$$\overset{\text{identity}}{=} \int_0^{\pi/4} \sqrt{\tan^2 x}\; dx$$

$$= \int_0^{\frac{\pi}{4}} |\tan x|\, dx$$

$$= \int_0^{\pi/4} \tan x \; dx \quad \text{as } \tan x \text{ is positive on } [0, \pi/4]$$

$$= \int_0^{\pi/4} \frac{\sin x}{\cos x}\, dx \quad , \quad \text{let } u = \cos x, \; du = -\sin x\, dx$$

$$= -\ln|\cos x| \Big|_0^{\pi/4}$$

$$= -\ln \cos\left(\frac{\pi}{4}\right) + \ln(\cos(0)|$$

$$= -\ln\left(\frac{\sqrt{2}}{2}\right) + \ln(1)$$

$$= -\ln \frac{\sqrt{2}}{2}$$

4. $\int 2y \sec^2(y) dy$ $= 2y \tan y - \int 2 \tan y \, dy$

using integration by parts

let $u = 2y$ $\quad dv = \sec^2 y \, dy$ $\qquad = 2y \tan y + 2 \ln|\cos y| + C$

$du = 2dy$ $\quad v = \tan y$

5. $\int e^{\theta} \sin(\theta) d\theta$ $= -e^{\theta} \cos\theta + \int \cos\theta \, e^{\theta} d\theta + C$

using integration by parts

let $u = e^{\theta}$ $\quad dv = \sin\theta \, d\theta$

$du = e^{\theta} d\theta$ $\quad v = -\cos\theta$ $\qquad = -e^{\theta}\cos\theta + e^{\theta}\sin\theta - \int \sin\theta \, e^{\theta} d\theta + C$

use integration by parts again

let $u = e^{\theta}$ $\quad dv = \cos\theta \, d\theta$

$du = e^{\theta} d\theta$ $\quad v = \sin\theta$

so we obtain that

$$\int e^{\theta} \sin\theta \, d\theta = -e^{\theta}\cos\theta + e^{\theta}\sin\theta - \int e^{\theta}\sin\theta \, d\theta + C$$



$$\Rightarrow \int e^{\theta} \sin\theta \, d\theta = \frac{1}{2}\left(-e^{\theta}\cos\theta + e^{\theta}\sin\theta\right) + C$$

6. $\int \frac{x+3}{2x(x-2)(x+2)}dx$ (Hint: use partial fraction decomposition.)

$$\frac{x+3}{2x(x-2)(x+2)} = \frac{A}{2x} + \frac{B}{x-2} + \frac{C}{x+2}$$

$$x+3 = Ax^2 - 4A + 2Bx^2 + 4Bx + 2Cx^2 - 4Cx$$

Equating coefficients

$$\left.\begin{aligned} 0 &= A + 2B + 2C \\ 1 &= 4B - 4C \\ 3 &= -4A \Rightarrow A = -\frac{3}{4} \end{aligned}\right\} \Rightarrow \begin{aligned} \frac{3}{4} &= 2B + 2C \\ 1 &= 4B - 4C \end{aligned}$$

$$\Rightarrow \begin{aligned} \frac{3}{2} &= 4B + 4C \\ 1 &= 4B - 4C \end{aligned} \Rightarrow \frac{5}{2} = 8B \Rightarrow B = \frac{5}{16} \Rightarrow \begin{aligned} 1 &= 4\left(\frac{5}{16}\right) - 4C \\ -\frac{1}{4} &= -4C \Rightarrow C = \frac{1}{16} \end{aligned}$$

So the integration becomes

$$\int \frac{-3/4}{2x}dx + \int \frac{5/16}{x-2}dx + \int \frac{1/16}{x+2}dx$$

$$= -\frac{3}{8}\ln|x| + \frac{5}{16}\ln|x-2| + \frac{1}{16}\ln|x+2| + C$$

7. $\int_{-\pi}^{\pi} \sin(3x)\sin(3x)dx$ by a trig identity this equals

$$= \frac{1}{2}\int_{-\pi}^{\pi} \cos(0)\,dx - \cos 6x\,dx$$

$$= \frac{1}{2}\int_{-\pi}^{\pi} 1\,dx - \cos 6x\,dx$$

$$= \frac{1}{2}\left[x - \frac{1}{6}\sin 6x\right]\Big|_{-\pi}^{\pi}$$

$$= \frac{1}{2}\left[\pi - \frac{1}{6}\sin 6\pi\right] - \frac{1}{2}\left[-\pi - \frac{1}{6}\sin(-6\pi)\right] = \pi$$

8. $\int 4\tan^3(\theta)d\theta$

$$= 4\int \tan^2\theta \cdot \tan\theta\, d\theta$$

$$= 4\int (\sec^2\theta - 1)\cdot \tan\theta\, d\theta$$

$$= 4\int \tan\theta \sec^2\theta - \tan\theta\, d\theta$$

Let $u = \tan\theta$

$du = \sec^2\theta\, d\theta$

$$= 4\left(\frac{\tan^2\theta}{2} + \ln|\cos\theta|\right) + C$$

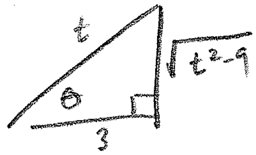
9. $\int \frac{3}{\sqrt{t^2-9}}dt$ $= \int \frac{9 \sec\theta \tan\theta \, d\theta}{\sqrt{9\sec^2\theta - 9}}$

let $t = 3\sec\theta$

$dt = 3\sec\theta \tan\theta \, d\theta$

$= 9\int \frac{\sec\theta \tan\theta \, d\theta}{\sqrt{9\tan^2\theta}} = 3\int \sec\theta \, d\theta$

$= 3 \ln|\sec\theta + \tan\theta| + C$



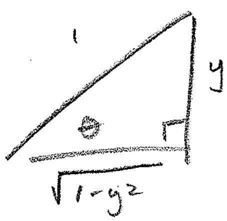

$= 3 \ln\left| \frac{t}{3} + \frac{\sqrt{t^2-9}}{3} \right| + C$

10. $\int \frac{4y^2}{(1-y^2)^{3/2}}dy$ $= \int \frac{4\sin^2\theta \cdot \cos\theta \, d\theta}{(\cos^2\theta)^{3/2}}$

let $y = \sin\theta$

$dy = \cos\theta \, d\theta$

$= \int \frac{4\sin^2\theta \cdot \cos\theta \, d\theta}{\cos^3\theta}$




$= \int 4\tan^2\theta \, d\theta$

$= 4\int \sec^2\theta - 1 \, d\theta$

$= 4\tan\theta - 4\theta + C$

$= 4\frac{y}{\sqrt{1-y^2}} - 4\sin^{-1}(y) + C$

Determine if the following integrals converge or diverge.

11. $\int_0^1 \frac{1}{(1+x)\sqrt{x}}dx$ (Hint: Let $u = \sqrt{x}$.)

The function is not continuous at zero, this is a Type II integral

$$= \lim_{b \to 0^+} \int_b^1 \frac{1}{(1+x)\sqrt{x}} dx \quad = \lim_{b \to 0^+} 2\int_{\sqrt{b}}^1 \frac{du}{1+u^2} \quad = \lim_{b \to 0^+} 2 \tan^{-1}(u) \Big|_{\sqrt{b}}^1$$

if $u = \sqrt{x}$ then $du = \frac{1}{2}x^{-1/2}dx$

$du = \frac{1}{2\sqrt{x}} dx$

$$= \lim_{b \to 0^+} 2\tan^{-1}(1) - 2\tan^{-1}(\sqrt{b})$$

$$= 2\frac{\pi}{4} - 0 = \frac{\pi}{2}$$

Converges.

12. $\int_2^\infty \frac{1}{x^{1.001}}dx$

a Type I integral

$$= \lim_{b \to \infty} \int_2^b \frac{1}{x^{1.001}} dx \quad = \lim_{b \to \infty} \frac{x^{-.001}}{-.001} \Big|_2^b$$

$$= \lim_{b \to \infty} \frac{b^{-.001}}{-.001} - \frac{2^{-.001}}{-.001}$$

$$= 0 + \frac{1}{.001(2)^{1/1000}}$$

$\Rightarrow$ converges

13. $\int_0^1 (x \ln x) dx$

The function is undefined at zero, so this is a Type II integral.

$$= \lim_{b \to 0^+} \int_b^1 x \ln x \, dx$$

using integration by parts with

$u = \ln x$ $\quad dv = x\,dx$

$du = \frac{1}{x} dx$ $\quad v = \frac{x^2}{2}$

we obtain that

$$= \lim_{b \to 0^+} \left. \frac{x^2}{2} \ln x \right|_b^1 - \int_b^1 \frac{x}{2} dx$$

$$= \lim_{b \to 0^+} \left. \frac{x^2}{2} \ln x - \frac{x^2}{4} \right|_b^1$$

$$= \lim_{b \to 0^+} \left( \frac{1}{2} \ln 1 - \frac{1}{4} \right) - \left( \frac{b^2}{2} \ln b - \frac{b^2}{4} \right)$$

$$= \frac{-1}{4} - \lim_{b \to 0^+} \frac{b^2}{2} \ln b - 0$$

$$= \frac{-1}{4} - \lim_{b \to 0^+} \frac{\ln b}{2 b^{-2}}$$

$$\overset{\text{L.H.}}{=} -\frac{1}{4} - \lim_{b \to 0^+} \frac{\frac{1}{b}}{-4b^{-3}} = -\frac{1}{4} - \lim_{b \to 0^+} \frac{-4b^2}{-4} = \frac{-1}{4}$$



$\Rightarrow$ converges.