Calculus II - Exam 1 - Spring 2013

March 7, 2013

Name:
Honor Code Statement:

Additional Honor Code Statement: I have not observed another violating the Honor Code._____________

Directions: Complete all problems. Justify all answers/solutions. Calculators are not permitted. Upon completing the exam, complete and sign the honor code and the additional statement given above.

1. [10 points] **Voltage in a discharging capacitor** Suppose that electricity is draining from a capacitor at a rate that is proportional to the voltage V across its terminals and that, if t is measured in scones,

$$\frac{dV}{dt} = -\frac{1}{40}V.$$

Solve this equation for V, using V_0 to denote the value of V when $t = 0$. How long will it take the voltage to drop to 10% of its original value? (As calculators are not allowed, leave the answer in terms of powers and logs.)

2. [5 points, each]Calculate the following limits. Identify the indeterminate form (if any).

- $\lim_{x\to 0} \frac{1}{\sin(x)} - \frac{1}{x}$

- $\lim_{x\to\infty} \frac{\ln(x+1)}{\log_2 x}$

3. [5 points, each] Differentiate the following functions with respect to x.

- $y = 7^{x^2-x+1}$

- $y = \ln(\sqrt{x})$

- $G(x) = \int_2^x \tan(t)\ln(t)dt$

4. [5 points, each] Evaluate the following integrals.

- $\int_0^4 2^x dx$

- $\int \frac{\sin(x)}{\cos(x)} dx$

5. Define/State:

- [5 points] State the Mean Value Theorem and draw a picture that helps illustrate the statement.

- [5 points] Give the statement of Theorem 7 of Section 6.1 and draw a picture that helps illustrate the statement.

6. [10 points] One of the Laws of Logarithms is: if x is a positive real number and r a rational number, then $\ln(x^r) = r\ln(x)$. Help finish the proof, a sketch of which is below, by filling in the blanks.

PROOF: Let $f(x) = \ln(x^r)$ and $g(x) = r\ln(x)$.

We have $f'(x) =$ __________________________

We also have $g'(x) =$ __________________________

Since these derivatives are the same, $f(x)$ and $g(x)$ differ by __________________________.
That is,

$$\ln(x^r) + C = r\ln(x).$$

Now, let $x = 1$. We get $\ln(1^r)+C = r\ln(1)$. So by the fact that __________________________
C equals __________________________.

Thus, $\ln(x^r) = r\ln(x)$.