Calculus II - Exam 1 - Spring 2012

March 6, 2012

Name:
Honor Code Statement:

Directions: Complete all problems. Justify all answers/solutions. Calculators, texts or notes are not permitted. The value of each problem is indicated in brackets. Please remember the writing expectations that we've discussed in class while keeping the time constraint in mind.

1. [5 points each] Differentiate the following functions with respect to x.

   - $y = \ln(\sin x)$

   - $y = e^{x^2+7x}$

   - $y = \log_8(\ln x)$

2. [8 points each] Calculate the following limits. Identify the indeterminate form (if any).

   - $\lim_{x\to 0} \frac{\cos(x)-1}{x^2}$

   - $\lim_{x\to 0}(e^x + x)^{\frac{1}{x}}$

3. [5 points each] Evaluate the following integrals.

- $\int_0^{\ln 16} e^{\frac{x}{4}} dx$

- $\int \frac{e^r}{1+e^r} dr$

4. [5 points each] Define/State:

   - State the Fundamental Theorem of Calculus, Part 1.

   - Define the Natural Logarithm Function as an area under a curve.

   - Use this definition to define the number e (Euler's constant).

5. [10 points] Use Theorem 7 of Section 6.1 of Stewart's text, the one we've talked so much about, to find the derivative of $y = \tan^{-1} x$. Justify the validity of each step.

- Use your result and the chain rule to find the derivative of $y = \tan^{-1} \sqrt{2x + 5}$.

6. [10 points] **The inversion of sugar** The processing of raw sugar has a step called *inversion* that changes the sugar's molecular structure. Once the process has begun, the rate of change of the amount of raw sugar is proportional to the amount of raw sugar remaining. If 1000 kg of raw sugar reduces to 800 kg of raw sugar during the first 10 hours, how much raw sugar will remain after another 14 hours? (As calculators are not allowed, leave the answer in terms of powers and logs.)

7. [8 points] One of the Laws of Exponents is that $e^{x+y} = e^x e^y$ for any real numbers x and y. A sketch of the proof is given below and you need to fill in some of the details.

First, $\ln(e^x e^y) = \ln(e^x) + \ln(e^y)$ since

Now $\ln(e^x) + \ln(e^y) = x + y$ since

And, $x + y = \ln(e^{x+y})$ since

Finally, we have shown that $\ln(e^x e^y) = \ln(e^{x+y})$. It follows that $e^x e^y = e^{x+y}$ since

8. [6 points] Suppose f is continuous, $f(0) = 0, f(1) = 1, f'(x) > 0$, and $\int_0^1 f(x)dx = \frac{1}{3}$. Find the value of the integral $\int_0^1 f^{-1}(y)dy$.