Calculus II - Exam 1 - Spring 2011

March 3, 2011

Name:
Honor Code Statement:

Directions: Complete all problems. Justify all answers/solutions. Calculators, texts or notes are not permitted. The value of each problem is indicated in brackets.

1. [10 points] **Reducing the cases of an infectious disease** We might model the way a disease dies out when properly treated by assuming that the rate dy/dt at which the number of infected people changes with respect to time t (in years) is proportional to the number of people with the disease y. The number of people cured is proportional to the number that have the disease. Suppose that in the course of any given year the number of cases of a disease is reduced by 10%. If there are $50,000$ cases today, how many years will it take to reduce the number to $1,000$? (As you don't have a calculator, I don't expect you to calculate this value as a decimal.)

2. [8 points] Suppose we wish to calculate the following limit,

$$\lim_{x \to 0} \frac{1 - \cos(x)}{x^2 + x}.$$

Below is an incorrect calculation, claiming that the limit is $1/2$. Identify the error in my calculation AND fix it.

$$\lim_{x \to 0} \frac{1 - \cos(x)}{x^2 + x} = \lim_{x \to 0} \frac{\sin(x)}{2x + 1} = \lim_{x \to 0} \frac{\cos(x)}{2} = \frac{1}{2}.$$

3. [2 points] **True or false?** We can always divide by e^x.

4. [5 points for each] Calculate each of the following:

-
$$\lim_{x \to \infty} x^{1/x}$$

-
$$G'(x) \quad where \quad G(x) = \int_{72}^{\ln(x)} t \; dt$$

- y' $where$ $y = e^x \sqrt{x} + \log_3(x)$

- $$\int_0^1 (x^5 + 5^x)dx$$

4

5. [5 points] **State** the definition of the natural logarithm as an integral.

6. [5 points] Why does the natural logarithm function have an inverse function?

7. [5 points] Find an equation of the tangent to the curve at the given point. $y = x \ln(x), \quad (e, e)$.

8. [10 points] Use Theorem 7 of Section 7.1 of Stewart's text, the one we've talked so much about, to find the derivative of $y = cos^{-1}x$. Justify, in words, the validity of each step.