Calculus II - Exam 1 - Fall 2013

October 3, 2013

Name:
Honor Code Statement:

Directions: Complete all problems. Justify all answers/solutions. Calculators, texts or notes are not permitted. The value of each problem is indicated in brackets. Please remember the writing expectations that we've discussed in class while keeping the time constraint in mind.

1. [10 points] Each of the following functions is continuous on their respective domains. These functions can be placed in order so that the knowledge of continuity of the first in the list helps imply the continuity of the second, the second imply the third, and so on. Let a be a positive real number. In no particular order, these functions are:
$$f(x) = \exp(x), g(x) = \log_a(x), h(x) = \frac{1}{x}, j(x) = \ln(x), k(x) = a^x.$$
Place the functions in the desired order.

2. [5 points] The simple form of L'Hopital's Rule states: If $f(a) = g(a) = 0$, $f'(a), g'(a)$ exist, and $g'(a) \neq 0$, then $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{f'(a)}{g'(a)}$. Since a picture can be worth a thousand words, sketch a figure or two that help illustrate this statement.

3. [5 points] Define what it means for a function to be **one-to-one**.

4. [10 points] Calculate the following limits. Identify the indeterminate form (if any).

- $\lim_{x \to 1^+} \ln(x) \tan(\pi x/2)$

- $\lim_{x \to 0} \frac{\tan(x) - x}{x^3}$

5. [10 points] Find the second derivative with respect to x of each of the following functions:

$$G(x) = \int_3^{x^2} 2^t \, dt$$

$$y = \ln(x \cdot e^{x^2})$$

6. [10 points] Evaluate the following integrals.

- $\int 3^{\sin(x)} \cos(x) dx$

- $\int \frac{\sec(x)\tan(x)}{2+3\sec(x)} dx$

7. [10 points] **Carbon 14** Carbon-14, one of the three isotopes of carbon, is radioactive and decays at a rate proportional to the amount present. Its half-life is 5730 years. If 10 grams were present originally, how much will be left after 2000 years? (As calculators are not permitted, you will leave your answer in terms of powers and logs.)

8. [10 points] One of the Laws of Exponents is that $e^{x-y} = \frac{e^x}{e^y}$ for any real numbers x and y. A sketch of the proof is given below and you need to fill in some of the details.

First, $\ln(\frac{e^x}{e^y}) = \ln(e^x) - \ln(e^y)$ since

Now $\ln(e^x) - \ln(e^y) = x - y$ since

And, $x - y = \ln(e^{x-y})$ since

Finally, we have shown that $\ln(\frac{e^x}{e^y}) = \ln(e^{x-y})$. It follows that $\frac{e^x}{e^y} = e^{x-y}$ since