Calculus I - Final Exam
Fall 2008

December 9, 2008

Name:
Honor Code Statement:

Directions: Complete all problems. Justify all answers/solutions. Calculators are not permitted, thus your solution to a problem need not be in decimal form. The point value of each problem is indicated in brackets.

1. [6 points] Find the inverse function and state its domain.

$$y = \ln(x + 3)$$

2. [3 points each] Give the following:

   (a) the integral definition of the natural logarithmic function,

   (b) the definition of e,

   (c) the derivative of the natural logarithmic function,

   (d) and the reason why its derivative is as such.

3. [5 points each] Differentiate each of the following functions:

(a) $f(x) = \sqrt{\ln(x)}$

(b) $g(x) = x \ln(x)$

(c) $y = \frac{e^x}{1+x}$

(d) $g(t) = e^{\sin^2 t}$

4. [6 points] Find an equation of the tangent line to the curve $y = 10^x$ at the point $(1, 10)$.

5. [5 points each] Evaluate each of the integrals:

(a) $\int \frac{\cos x}{2+\sin x} dx$

(b) $\int_0^5 e^{-3x} dx$. And, what theorem are you using to compute this?

(c) $\int \frac{\log_6(x)}{x} dx$

6. [4 points each] Bismuth-210 has a half-life of 5 days.

   (a) A sample originally has a mass of $800mg$. Find a formula for the mass remaining after t days.

   (b) Find the mass remaining after 30 days.

   (c) When is the mass reduced to $1mg$?

   (d) Give an expression for the average mass of the sample over the first 30 days.

7. [6 points] State the limit definition of the derivative and use this definition to find the derivative of $f(x) = 4x^2 - 10$.

8. [6 points] If $f(x) = x^3 - x$, does there exist a number c between 0 and 2 such that $f'(c) = \frac{f(2)-f(0)}{2}$? Why, or why not?

9. [8 points] Find the x-coordinate of the point on the line $y = 2x + 2$ that is closest to the origin.

10. [5 points] Find the area of the region bounded by the curves $y = 3^x, y = 4^x, x = -1, x = 1$.