Calculus I - Exam 3
Fall 2008

November 18, 2008

Name:
Honor Code Statement:

Directions: Complete all problems. Justify all answers/solutions. Calculators are not permitted. The point value of each problem is indicated in brackets.

1. [5 points] Find the derivative of the following function.

$$F(x) = \int_3^x \sin(t)\cos(t)dt$$

2. [5 points] Water flows from a tank at a rate of $r(t) = 200 - 4t$ liters per minute, where $0 \le t \le 50$. Find the amount of water that flows from the tank during the first 10 minutes.

3. [5 points each] Evaluate the indefinite integral.

(a) $\int (5\cos x + \sec x \tan x + x^4)\ dx$

(b) $\int (1 + \tan(\theta))^5 \sec^2(\theta)\ d\theta$

(c) $\int x^2 \sin(x^3)\ dx$

4. [5 points] Why is the following statement false?

$$\int_{-2}^{1} \frac{1}{x^4} dx = \frac{-3}{8}$$

5. [5 points] Suppose a particle moves back and forth along a straight line with velocity $v(t)$, measured in feet per second.

   (a) What is the meaning of $\int_{60}^{120} v(t) dt$?

   (b) What is the meaning of $\int_{60}^{120} |v(t)| dt$?

6. [10 points] Sketch the region enclosed by $y = \sqrt{x}, y = \frac{1}{2}x$ and $x = 9$. Then find the area of the region.

7. [10 points] Use the Principle of Mathematical Induction to prove that for each natural number n the following holds:

$$5 + 7 + 9 + \cdots + (2n + 3) = n(n + 4).$$

8. [10 points] Find the numbers b such that the average value of $f(x) = 2 + 6x - 3x^2$ on the interval $[0, b]$ is equal to 3.

9. [10 points] Find (but do not evaluate) an expression for the volume of the solid obtained by rotating the region bounded by $y = \frac{1}{x}, y = 0, x = 1$ and $x = 3$ about $y = -1$. Sketch the region that is being rotated.