1. [6 points] Find the absolute maximum and absolute minimum values of \( f(x) = x^3 - 3x + 1 \) on \([0,3]\).

2. [6 points] What does it mean when we write \( \lim_{x \to \infty} f(x) = L \)?
3. Consider the following function and find each of the following listed items. Upon completing each of the steps [2 points each], sketch the curve [6 points] on the graph paper provided.

\[ f(x) = 1 - \frac{3}{x^2 + 3} \]

(a) Determine the domain of \( f(x) \).

(b) Determine the \( y \)-intercept of \( f(x) \).

(c) Determine any \( x \)-intercepts of \( f(x) \).

(d) Determine if the graph is symmetric about the \( y \)-axis.

(e) Determine if the graph is symmetric about the origin.
(f) Determine any horizontal asymptotes of $f(x)$.

(g) Determine any vertical asymptotes of $f(x)$.

(h) Determine intervals of increase and decrease.
(i) Determine any local extrema.

(j) Determine intervals of concavity.

(k) Determine any inflection points.
4. [12 points] Find the dimensions of a rectangle with area 1,000\(m^2\) whose perimeter is as small as possible.
5. [6 points] Find all numbers $c$ that satisfy the conclusion of the Mean Value Theorem when $f(x) = x^3 + x - 1$ and we consider the interval $[0, 2]$. 
Theorem 1 [Rolle’s Theorem] Let $f$ be a function that satisfies the following three hypotheses:

(a) $f$ is continuous on the closed interval $[a, b]$.
(b) $f$ is ........ on the open interval $(a, b)$.
(c) $f(a) = ........$

Then there is a number $c$ in $(a, b)$ such that ...........

Proof of the case $f(x) > f(a)$ for some $x \in (a, b)$:

By the ........ Theorem (which we can apply by the first hypothesis), $f$ has a maximum value somewhere in $[a, b]$. By the third hypothesis, it must attain this maximum value at a number $c$ in the open interval $(a, b)$. Then $f$ has a ........ at $c$ and, by the second hypothesis, $f$ is differentiable at $c$. Therefore, the conclusion holds by ........ Theorem.