Calculus I - Exam 2
Fall 2008

October 28, 2008

Name:
Honor Code Statement:

Directions: Complete all problems. Justify all answers/solutions. Calculators are not permitted. The point value of each problem is indicated in brackets.

1. [6 points] Find the absolute maximum and absolute minimum values of $f(x) = x^3 - 3x + 1$ on $[0, 3]$.

2. [6 points] What does it mean when we write $\lim_{x\to\infty} f(x) = L$?

3. Consider the following function and find each of the following listed items. Upon completing each of the steps [2 points each], sketch the curve [6 points] on the graph paper provided.

$$f(x) = 1 - \frac{3}{x^2 + 3}$$

(a) Determine the domain of $f(x)$.

(b) Determine the y-intercept of $f(x)$.

(c) Determine any x-intercepts of $f(x)$.

(d) Determine if the graph is symmetric about the y-axis.

(e) Determine if the graph is symmetric about the origin.

(f) Determine any horizontal asymptotes of $f(x)$.

(g) Determine any vertical asymptotes of $f(x)$.

(h) Determine intervals of increase and decrease.

(i) Determine any local extrema.

(j) Determine intervals of concavity.

(k) Determine any inflection points.

4. [12 points] Find the dimensions of a rectangle with area $1,000m^2$ whose perimeter is as small as possible.

5. [6 points] Find all numbers c that satisfy the conclusion of the Mean Value Theorem when $f(x) = x^3 + x - 1$ and we consider the interval $[0, 2]$.

6. Given below is an incomplete statement and sketch of a particular case in the proof of Rolle's theorem. Your task is to complete the sketch by filling in the blanks. There are 6 blanks to fill in [2 points each].

Theorem 1 *[Rolle's Theorem] Let f be a function that satisfies the following three hypotheses:*

(a) f is continuous on the closed interval $[a, b]$.

(b) f is .... on the open interval (a, b).

(c) $f(a) = \dots\dots\dots.$.

Then there is a number c in (a, b) such that ....

PROOF OF THE CASE $f(x) > f(a)$ FOR SOME $x \in (a, b)$:

By the .... Theorem (which we can apply by the first hypothesis), f has a maximum value somewhere in $[a, b]$. By the third hypothesis, it must attain this maximum value at a number c in the open interval (a, b). Then f has a .... at c and, by the second hypothesis, f is differentiable at c. Therefore, the conclusion holds by .... Theorem.