Calculus I - Exam 1
Fall 2008

October 7, 2008

Name:
Honor Code Statement:

Directions: Complete all problems. Justify all answers/solutions. Calculators are not permitted. The point value of each problem is indicated in brackets.

1. [5 points each] Find the derivative of each of the following functions. You may use any method you wish. [1 point each] **STATE** the rule or method you use for each problem.

   (a) $y = x^3 - 10x^2 + 17$

   (b) $g(t) = \sqrt{(sin(t))}$

(c) $F(x) = x^2 \cos x$

(d) $xy^4 + x^2y = x + 3y$

2. [8 points] Find an equation of the tangent line to the curve at the given point.

$$y = \frac{x^2 + 1}{x^2 - 1}, \qquad (0, -1)$$

[3 points] Does this function have any discontinuities? If so, where? If so, what type?

3. [8 points] If a stone is thrown vertically upward from the surface of the moon with a velocity of $10m/s$, its height (in meters) after t seconds is $h = 10t - 0.83t^2$. What is the velocity of the stone after $3s$?

4. [6 points] Is there a root of the equation in the given interval? Why, or why not?

$$2x^3 + x^2 + 2 = 0, \qquad (-2, -1)$$

5. [4 points] Does $\lim_{x\to a}\cos(x+\sin x)=\cos(\lim_{x\to a}(x+\sin x))$? Why, or why not?

6. [8 points] Prove that $\lim_{x\to 3}(6x-2)=16$ using the ϵ,δ definition of limit.

7. [8 points] The length of a rectangle is increasing at a rate of $8cm/s$ and its width is increasing at rate of $3cm/s$. When the length is $20cm$ and the width is $10cm$, how fast is the area of the rectangle increasing?

8. Given below is an incomplete sketch of the proof of the following theorem: *If f is differentiable at a, the f is continuous at a.* Your task is to complete the sketch. For each of the **starred *** equality signs [2 points each], give a justification as to why the step is valid (that is, give reason why what is on the previous side of the equal sign is, in fact, equal to that which follows). There are two blanks to fill in [2 points each].

PROOF:

To show that f is continuous at a, we must, by the definition, show that $\lim_{x\to a} f(x)$ equals . . . . (Fill in the blank.)

Thus, we consider

$\lim_{x\to a} f(x) = \lim_{x\to a} f(a) + f(x) - f(a)$,

by a clever trick of adding zero,

$=^* \lim_{x\to a} f(a) + \lim_{x\to a} f(x) - f(a)$,

$=^* \lim_{x\to a} f(a) + \lim_{x\to a} [(f(x) - f(a))\frac{x-a}{x-a}]$,

$=^* \lim_{x\to a} f(a) + \lim_{x\to a} \frac{f(x)-f(a)}{x-a} \lim_{x\to a} x - a$,

$=^* \lim_{x\to a} f(a) + f'(a) \cdot 0,$

$=^* \lim_{x\to a} f(a),$

$= \ldots\ldots\ldots$ Fill in the blank.

This completes the proof.