

FORMULAS PROVIDED ON EXAM 3

Change of coordinates

Cylindrical to Cartesian:

$$x = r \cos \theta, \quad y = r \sin \theta, \quad z = z$$

Cartesian to cylindrical:

$$r^2 = x^2 + y^2, \quad \tan(\theta) = \frac{y}{x}, \quad z = z$$

Spherical to Cartesian:

$$x = \rho \sin \varphi \cos \theta, \quad y = \rho \sin \varphi \sin \theta, \quad z = \rho \cos \varphi$$

Cartesian to spherical:

$$\rho^2 = x^2 + y^2 + z^2, \quad \tan(\varphi) = \sqrt{x^2 + y^2}/z, \quad \tan(\theta) = \frac{y}{x}$$

Spherical to cylindrical:

$$r = \rho \sin(\varphi), \quad \theta = \theta, \quad z = \rho \cos(\varphi)$$

Cylindrical to spherical:

$$\rho^2 = r^2 + z^2, \quad \tan(\varphi) = r/z, \quad \theta = \theta$$

Change of variables in triple integrals:

$$\int \int \int_W f(x, y, z) dx dy dz = \int \int \int_{W^*} f(x(u, v, w), y(u, v, w), z(u, v, w)) \left| \frac{\partial(x, y, z)}{\partial(u, v, w)} \right| du dv dw$$

Volume elements:

$$dV = dx \, dy \, dz \text{ Cartesian}$$

$$dV = r \, dr \, d\theta \, dz \text{ Cylindrical}$$

$$dV = \rho^2 \sin \varphi \, d\rho \, d\varphi \, d\theta \text{ spherical}$$

$$dV = \left| \frac{\partial(x, y, z)}{\partial(u, v, w)} \right| du \, dv \, dw \text{ general}$$

Trigonometric Identities

Addition and subtraction formulas

- $\sin(x + y) = \sin x \cos y + \cos x \sin y$
- $\sin(x - y) = \sin x \cos y - \cos x \sin y$
- $\cos(x + y) = \cos x \cos y - \sin x \sin y$
- $\cos(x - y) = \cos x \cos y + \sin x \sin y$
- $\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$
- $\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

Double-angle formulas

- $\sin(2x) = 2 \sin x \cos x$
- $\cos(2x) = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x$
- $\tan(2x) = \frac{2 \tan x}{1 - \tan^2 x}$

Half-angle formulas

- $\sin^2 x = \frac{1 - \cos(2x)}{2}$
- $\cos^2 x = \frac{1 + \cos(2x)}{2}$

Others

- $\sin A \cos B = \frac{1}{2} [\sin(A - B) + \sin(A + B)]$
- $\sin A \sin B = \frac{1}{2} [\cos(A - B) - \cos(A + B)]$
- $\cos A \cos B = \frac{1}{2} [\cos(A - B) + \cos(A + B)]$

Pythagorean and reciprocal identities

- If you don't know these, then get a tattoo.