MULTIVARIABLE CALCULUS
EXAM 3
FALL 2018

Name:
Honor Code Statement:

Directions: Complete all problems. Justify all answers/solutions. Electronic devices, books, and notes are not permitted. Please turn off cell phones and other devices; these should not be used under any circumstances. The last two pages contains formulas. Best of luck.

(1) [10 points] Compute the following iterated integral:

$$\int_{-2}^{3} \int_{0}^{1} (|x| \sin(\pi y)) \, dy \, dx$$

Date: December 11, 2018.

(2) [10 points] Evaluate $\iint_D e^{x^2}\,dA$, where D is the triangular region with vertices $(0,0)$, $(1,0)$ and $(1,1)$. Identify whether D is a Type I, Type II or Type III elementary region.

(3) [5 points] Let f be a continuous function on an elementary region D of the plane. Define the *extension of* f. In two sentences, what purpose does this definition serve?

(4) [5 points] Calculate the scalar line integral of the function $f(x, y, z) = x^2 z$ over the path $\mathbf{x}(t) = (3t, 2t, 3t), 0 \leq t \leq 1$.

(5) [10 points] Write the following triple integral

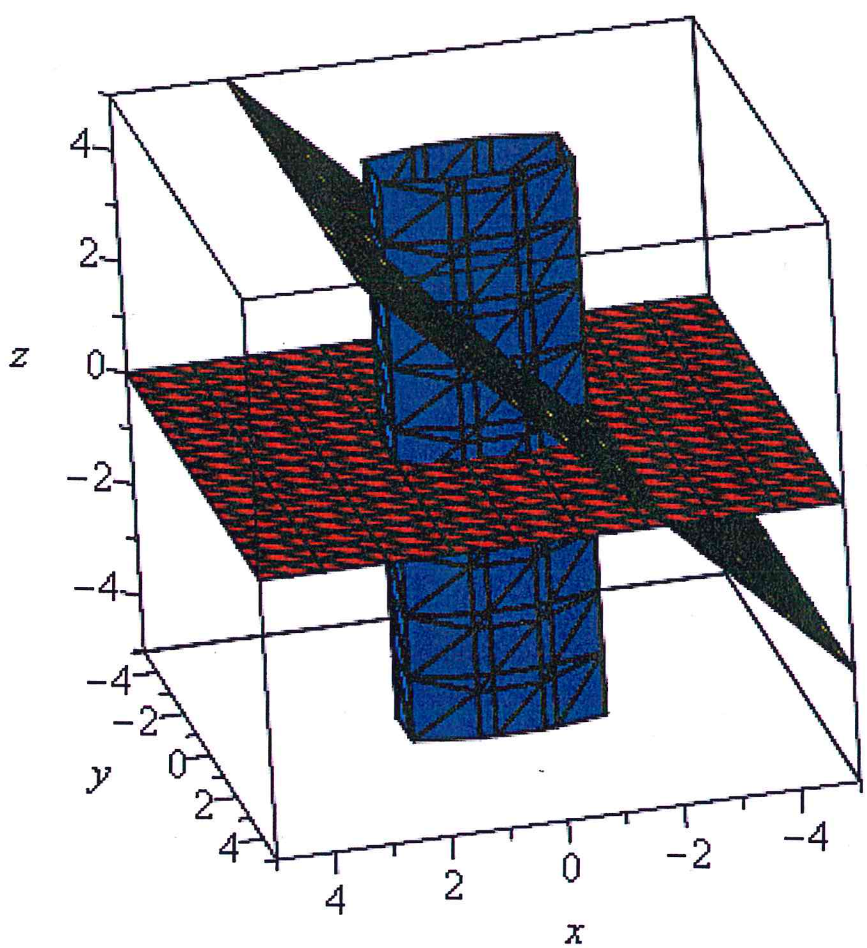
$$\iiint\limits_W z\, dV,$$

where W is the region bounded by $z = 0$, $x^2 + 4y^2 = 4$ and $z = x + 2$, as an iterated integral with order of integration z, then y, then x. You need not compute the integral. A sketch of the region W as given by Maple is provided. What theorem allows us to do this? and what properties does the integrand z have to justify its application?

> with(plots);

[animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d, conformal, conformal3d, contourplot, contourplot3d, coordplot, coordplot3d, densityplot, display, dualaxisplot, fieldplot, fieldplot3d, gradplot, gradplot3d, implicitplot, implicitplot3d, inequal, interactive, interactiveparams, intersectplot, listcontplot, listcontplot3d, listdensityplot, listplot, listplot3d, loglogplot, logplot, matrixplot, multiple, odeplot, pareto, plotcompare, pointplot, pointplot3d, polarplot, polygonplot, polygonplot3d, polyhedra_supported, polyhedraplot, rootlocus, semilogplot, setcolors, setoptions, setoptions3d, shadebetween, spacecurve, sparsematrixplot, surfdata, textplot, textplot3d, tubeplot] **(1)**

> implicitplot3d($[z = 0, x^2 + 4 \cdot y^2 = 4, z = x + 2], x = -5..5, y = -5..5, z = -5..5$, color = [red, blue, yellow]);



>

(6) [10 points] (from Stewart) Evaluate

$$\iint_R (3x + 4y^2)\, dA,$$

where R is the region in the upper half-plane bounded by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

(7) Suppose that C is the curve $y = f(x)$, oriented from $(a, f(a))$ to $(b, f(b))$ where $a < b$ and where f is positive and continuous on $[a, b]$. If $\mathbf{F} = y\mathbf{i}$, show that the value of

$$\int_C \mathbf{F} \cdot d\mathbf{s}$$

is the area under the graph of f between $x = a$ and $x = b$.

(8) [10 points] Let f and g be single-variable functions of class C^1. Let C be a piecewise C^1, simple, closed curve in $\mathbb{R}^2$. Compute

$$\oint_C f(x)dx + g(y)dy.$$

Justify your solution.

Change of coordinates

Cylindrical to Cartesian:

$$x = r\cos\theta,\ y = r\sin\theta, z = z$$

Cartesian to cylindrical:

$$r^2 = x^2 + y^2,\ \tan(\theta) = \frac{y}{x},\ z = z$$

Spherical to Cartesian:

$$x = \rho\sin\varphi\cos\theta,\ y = \rho\sin\varphi\sin\theta,\ z = \rho\cos\varphi$$

Cartesian to spherical:

$$\rho^2 = x^2 + y^2 + z^2,\ \tan(\varphi) = \sqrt{x^2+y^2}/z,\ \tan(\theta) = \frac{y}{x}$$

Spherical to cylindrical:

$$r = \rho\sin(\varphi),\ \theta = \theta,\ z = \rho\cos(\varphi)$$

Cylindrical to spherical:

$$\rho^2 = r^2 + z^2,\ \tan(\varphi) = r/z,\ \theta = \theta$$

Change of variables in triple integrals:

$$\int\int\int_W f(x,y,z)dxdydz = \int\int\int_{W^*} f(x(u,v,w), y(u,v,w), z(u,v,w))|\frac{\partial(x,y,z)}{\partial(u,v,w)}|dudvdw$$

Volume elements:

$$dV = dx\ dy\ dz\ Cartesian$$
$$dV = r\ dr\ d\theta\ dz\ Cylindrical$$
$$dV = \rho^2\sin\varphi\ d\rho\ d\varphi\ d\theta\ spherical$$
$$dV = |\frac{\partial(x,y,z)}{\partial(u,v,w)}|du\ dv\ dw\ general$$

Trigonometric Identities

Addition and subtraction formulas

- $\sin(x+y) = \sin x \cos y + \cos x \sin y$
- $\sin(x-y) = \sin x \cos y - \cos x \sin y$
- $\cos(x+y) = \cos x \cos y - \sin x \sin y$
- $\cos(x-y) = \cos x \cos y + \sin x \sin y$
- $\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$
- $\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

Double-angle formulas

- $\sin(2x) = 2\sin x \cos x$
- $\cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$
- $\tan(2x) = \frac{2\tan x}{1-\tan^2 x}$

Half-angle formulas

- $\sin^2 x = \frac{1-\cos(2x)}{2}$
- $\cos^2 x = \frac{1+\cos(2x)}{2}$

Others

- $\sin A \cos B = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$
- $\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$
- $\cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$

Pythagorean and reciprocal identities

- If you don't know these, then get a tattoo.