MULTIVARIABLE CALCULUS
EXAM 3
FALL 2014

Name:
Honor Code Statement:

Directions: Complete all problems. Each problem is worth 10 points. Justify all answers/solutions. Electronic devices, books, and notes are not permitted. Please turn off cell phones and other devices. The last two pages contains formulas. Best of luck.

(1) Evaluate $\int\int_D (x^2 + y^2) dA$, where D is the region in the first quadrant bounded by $y = x, y = 3x$ and $xy = 3$. (Hint: two pieces.)

Date: December 10, 2014.

(2) Rewrite the given sum of iterated integrals as a single iterated integral by reversing the order of integration, and evaluate.

$$\int_0^1 \int_0^x \sin(x)dy\ dx + \int_1^2 \int_0^{2-x} \sin(x)dy\ dx$$

(3) Integrate the given function over the indicated region W: $f(x,y,z) = 1-z^2$; W is the tetrahedron with vertices $(0,0,0),(1,0,0),(0,2,0)$, and $(0,0,3)$.

(4) Transform the given integral in Cartesian coordinates to one in polar coordinates and evaluate the polar integral.

$$\int_{-a}^{a}\int_{0}^{\sqrt{a^2-y^2}} e^{x^2+y^2}\,dx\;dy$$

(5) Calculate $\int_{\mathbf{x}} f\, ds$, where $f(x,y,z) = 3x+xy+z^3$, $\mathbf{x}(t) = (\cos(4t), \sin(4t), 3t)$, $0 \leq t \leq 2\pi$.

(6) Find $\int_{\mathbf{x}} \mathbf{F} \cdot d\mathbf{s}$, where $\mathbf{F} = x\mathbf{i} + y\mathbf{j}, \mathbf{x}(t) = (2t+1, t+2),\ 0 \le t \le 1.$

(7) Use Green's theorem to find the area between the ellipse $x^2/9 + y^2/4 = 1$ and the circle $x^2 + y^2 = 25$. (Hint: a parametrization for the ellipse is $(3\cos(t), -2\sin(t))$.)

Change of coordinates

Cylindrical to Cartesian:

$$x = r\cos\theta,\ y = r\sin\theta, z = z$$

Cartesian to cylindrical:

$$r^2 = x^2 + y^2,\ \tan(\theta) = \frac{y}{x},\ z = z$$

Spherical to Cartesian:

$$x = \rho\sin\varphi\cos\theta,\ y = \rho\sin\varphi\sin\theta,\ z = \rho\cos\varphi$$

Cartesian to spherical:

$$\rho^2 = x^2 + y^2 + z^2,\ \tan(\varphi) = \sqrt{x^2+y^2}/z,\ \tan(\theta) = \frac{y}{x}$$

Spherical to cylindrical:

$$r = \rho\sin(\varphi),\ \theta = \theta,\ z = \rho\cos(\varphi)$$

Cylindrical to spherical:

$$\rho^2 = r^2 + z^2,\ \tan(\varphi) = r/z,\ \theta = \theta$$

Change of variables in triple integrals:

$$\int\int\int_W f(x,y,z)dxdydz = \int\int\int_{W^*} f(x(u,v,w), y(u,v,w), z(u,v,w))|\frac{\partial(x,y,z)}{\partial(u,v,w)}|dudvdw$$

Volume elements:

$$dV = dx\ dy\ dz\ Cartesian$$
$$dV = r\ dr\ d\theta\ dz\ Cylindrical$$
$$dV = \rho^2\sin\varphi\ d\rho\ d\varphi\ d\theta\ spherical$$
$$dV = |\frac{\partial(x,y,z)}{\partial(u,v,w)}|du\ dv\ dw\ general$$

Trigonometric Identities

Addition and subtraction formulas

- $\sin(x+y) = \sin x \cos y + \cos x \sin y$
- $\sin(x-y) = \sin x \cos y - \cos x \sin y$
- $\cos(x+y) = \cos x \cos y - \sin x \sin y$
- $\cos(x-y) = \cos x \cos y + \sin x \sin y$
- $\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$
- $\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

Double-angle formulas

- $\sin(2x) = 2 \sin x \cos x$
- $\cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$
- $\tan(2x) = \frac{2 \tan x}{1 - \tan^2 x}$

Half-angle formulas

- $\sin^2 x = \frac{1 - \cos(2x)}{2}$
- $\cos^2 x = \frac{1 + \cos(2x)}{2}$

Others

- $\sin A \cos B = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$
- $\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$
- $\cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$

Pythagorean and reciprocal identities

- If you don't know these, then get a tattoo.