MULTIVARIABLE CALCULUS
EXAM 3
FALL 2013

Name:
Honor Code Statement:

Directions: Complete all problems. Justify all answers/solutions. Electronic devices, books, and notes are not permitted. Please turn off cell phones and other devices. The last two pages contains formulas. Best of luck.

(1) [10 points] Calculate the given iterated integral and indicate of what region in $\mathbb{R}^3$ it may be considered to represent the volume.

$$\int_{-2}^{3}\int_{0}^{1} |x| sin(\pi y)\ dy\ dx$$

Date: December 10, 2013.

(2) [10 points] Let D be the region bounded by $x = y^3$ and $y = x^2$. We are interested in computing $\iint_D xy \ dA$. Give an equivalent iterated integral where you treat D as a Type I elementary region. Then do the same when you treat D as a Type II elementary region. Finally, compute a numerical value for one of these iterated integrals.

(3) [10 points] Show how to set up the following triple integral as an iterated integral in one-half of the total number of ways possible, but do NOT evaluate the integral. Find $\iiint_W z\ dV$, where W is the region in the first octant bounded by the cylinder $y^2 + z^2 = 9$ and the planes $y = x, x = 0$, and $z = 0$. (A Maple sketch of W is included with the exam packet.)

(4) [10 points] Transform the given integral in Cartesian coordinates to one in polar coordinates and evaluate the polar integral. Give a sketch of D and D^*. **Also,** find the area of D and the area of D^* and state the relation between these two areas.

$$\int_0^2 \int_0^{\sqrt{4-x^2}} dy\, dx$$

(5) [10 points] Verify Green's theorem for the vector field $\mathbf{F} = 2y\mathbf{i} + x\mathbf{j}$, and D is the semicircular region $x^2 + y^2 \leq a^2$ and $y \geq 0$ by calculating both $\oint_{\delta D} M\ dx + N\ dy$ and $\int\int_D (N_x - M_y)\ dA$.

(6) [5 points] Calculate the following difference of two scalar line integrals, $\int_{\mathbf{x}} f\ ds - \int_{\mathbf{y}} g\ ds$, where $f(x, y, z) = x + y + z$ and $g(x, y, z) = z + y + x$, and

$$\mathbf{x}(t) = \begin{cases} (2t, 0, 0) & : \text{if } 0 \leq t \leq 1 \\ (2, 3t - 3, 0) & : \text{if } 1 \leq t \leq 2 \\ (2, 3, 2t - 4) & : \text{if } 2 \leq t \leq 3 \end{cases}$$

$$\mathbf{y}(t) = \begin{cases} (4t, 0, 0) & : \text{if } 0 \leq t \leq 1/2 \\ (2, 6t - 3, 0) & : \text{if } 1/2 \leq t \leq 1 \\ (2, 3, \frac{1}{50}t - \frac{1}{50}) & : \text{if } 1 \leq t \leq 101 \end{cases}$$

Change of coordinates

Cylindrical to Cartesian:

$$x = r\cos\theta,\ y = r\sin\theta, z = z$$

Cartesian to cylindrical:

$$r^2 = x^2 + y^2,\ \tan(\theta) = \frac{y}{x},\ z = z$$

Spherical to Cartesian:

$$x = \rho\sin\varphi\cos\theta,\ y = \rho\sin\varphi\sin\theta,\ z = \rho\cos\varphi$$

Cartesian to spherical:

$$\rho^2 = x^2 + y^2 + z^2,\ \tan(\varphi) = \sqrt{x^2 + y^2}/z,\ \tan(\theta) = \frac{y}{x}$$

Spherical to cylindrical:

$$r = \rho\sin(\varphi),\ \theta = \theta,\ z = \rho\cos(\varphi)$$

Cylindrical to spherical:

$$\rho^2 = r^2 + z^2,\ \tan(\varphi) = r/z,\ \theta = \theta$$

Change of variables in triple integrals:

$$\int\int\int_W f(x,y,z)dxdydz = \int\int\int_{W^*} f(x(u,v,w), y(u,v,w), z(u,v,w))|\frac{\partial(x,y,z)}{\partial(u,v,w)}|dudvdw$$

Volume elements:

$$dV = dx\ dy\ dz\ Cartesian$$
$$dV = r\ dr\ d\theta\ dz\ Cylindrical$$
$$dV = \rho^2\sin\varphi\ d\rho\ d\varphi\ d\theta\ spherical$$
$$dV = |\frac{\partial(x,y,z)}{\partial(u,v,w)}|du\ dv\ dw\ general$$

Trigonometric Identities

Addition and subtraction formulas

- $\sin(x+y) = \sin x \cos y + \cos x \sin y$
- $\sin(x-y) = \sin x \cos y - \cos x \sin y$
- $\cos(x+y) = \cos x \cos y - \sin x \sin y$
- $\cos(x-y) = \cos x \cos y + \sin x \sin y$
- $\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$
- $\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

Double-angle formulas

- $\sin(2x) = 2 \sin x \cos x$
- $\cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$
- $\tan(2x) = \frac{2 \tan x}{1 - \tan^2 x}$

Half-angle formulas

- $\sin^2 x = \frac{1 - \cos(2x)}{2}$
- $\cos^2 x = \frac{1 + \cos(2x)}{2}$

Others

- $\sin A \cos B = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$
- $\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$
- $\cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$

Pythagorean and reciprocal identities

- If you don't know these, then get a tattoo.