MULTIVARIABLE CALCULUS
EXAM 2
SPRING 2024

Name:
Honor Code Statement:

Directions: Complete all problems. Justify all answers/solutions. Each problem is worth 10 points. Calculators/notes/texts/cell-phones are not permitted – the only permitted item is a writing utensil. Best of luck.

(1) The position vector of an object moving in a plane is given by $\mathbf{x}(t) = t^3\mathbf{i} + t^2\mathbf{j}$. Find its velocity, speed, and acceleration when $t = 1$. Give an appropriate sketch of the path for $t \geq 0$. Then find an equation for the line tangent to $\mathbf{x}$ at $\mathbf{x}(1)$.

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(2) Set up the correct definite integral for computing the length of the given path. You do not need to solve it.

$$\mathbf{x}(t) = (\ln(\cos(t)), \cos(t), \sin(t)),\ \frac{\pi}{6} \leq t \leq \frac{t}{3}$$

(3) Calculate the flow line $\mathbf{x}(t)$ of the vector field $\mathbf{F}(x, y) = (x^2, y)$ at the point $\mathbf{x}(1) = (1, e)$. Sketch this flow line starting at $t = 1$. Calculate the divergence of $\mathbf{F}$ at this point.

(4) Find the first- and second-order Taylor polynomials for the function

$$f(x, y) = e^x \sin(y)$$

at the point $\mathbf{a} = (1, \frac{\pi}{2})$.

(5) Use the second derivative test for functions of two variables to determine the nature of the critical points of the following function which I'm telling you are $(0,0)$ and $(2,0)$.

$$f(x,y) = e^{-x}(x^2 + 3y^2)$$

(6) Use the method of Lagrange multipliers to find the extreme values of the function $f(x, y) = x^2 + 2y^2$ subject to the condition $x^2 + y^2 = 1$.

Write the system of equations here.

Note that the first equation gives $x = 0$ or $\lambda = 1$. Use this hint to find the critical points.

Evaluate these critical points on the function to determine which correspond to maximum and which to minimum.