MULTIVARIABLE CALCULUS
EXAM 2
SPRING 2021

Name:
Honor Code Statement:

Directions: Complete all problems. Justify all answers/solutions. Each problem is worth 10 points. Calculators/notes/texts/cell-phones are not permitted – the only permitted item is a writing utensil. Best of luck.

(1) Sketch the image of the following path, using arrows to indicate the direction in which the parameter increases:

$$\mathbf{x}(t) = e^t\mathbf{i} + e^{-t}\mathbf{j}$$

.

Now find an equation for the line tangent to this path at $t = 1$.

Date: April 22, 2021.

(2) Consider the curve $g(t) = (t, \frac{2}{3}t^{3/2})$. Give the arc length $s(t)$ in terms of the parameter t measured from 0. Then solve for t in terms of s. Next give the function with uniform speed 1 which traces the same curve.

(3) Describe to me in words the gradient field of the following function,

$$g(x, y, z) = \frac{1}{4}(x^2 + y^2 + z^2).$$

Your response should include use of the words “direction” and “magnitude”.

(4) If $\mathbf{F}(x, y, z) = (y, z^2, x^3)$, then describe the vectors of the vector field $curl\mathbf{F}$ that lie along the y-axis (i.e. where $x = z = 0$). Your response should include use of the words "direction" and "magnitude".

(5) Find the first- and second-order Taylor polynomials for the function

$$f(x,y) = ye^{3x} + e^{2y}$$

at the point $\mathbf{a} = (0,2)$.

(6) Consider the function $f(x, y) = x^2 + y^2$ for points (x, y) in the square $U = \{(x, y) | -1 \leq x \leq 1, -1 \leq y \leq 1\}$. Find the critical points of f. Determine the nature of these critical points. Does f have a global maximum on U? If yes, suggest where it might be. If no, explain why not.

(7) Set up (but do not solve) the Lagrange equations that would allow one to determine the maximum value of $f(x, y, z) = x - y + z$, subject to the condition $x^2 + y^2 + z^2 = 1$.