MULTIVARIABLE CALCULUS
EXAM 2
SPRING 2018

Name:
Honor Code Statement:

Directions: Complete all problems. Justify all answers/solutions. Computational problems are worth 10 points each; others are worth 5 points each. Calculators/notes/texts/cellphones are not permitted – the only permitted item is a writing utensil. Best of luck.

(1) Find an equation for the line tangent to the path $\mathbf{x}(t) = (\cos(e^t), 3 - t^2, t)$ at the value $t = 1$.

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(2) Find the length of the path $\mathbf{x}(t) = (t^3, 3t^2, 6t)$ for $-1 \leq t \leq 2$.

(3) Define *vector field*.

(4) Some of these are scalar fields and some are vector fields, say what each is.

- grad f
- div $\mathbf{F}$
- curl $\mathbf{F}$
- grad $f \times$ div $\mathbf{F}$

(5) Calculate the flow line $\mathbf{x}(t)$ of the vector field $\mathbf{F}(x, y) = x\mathbf{i} - y\mathbf{j}$ at the point $\mathbf{x}(0) = (1, 2)$.

(6) Given $\mathbf{F}(x, y, z) = xyz\mathbf{i} - e^z \cos(x)\mathbf{j} + xy^2z^3\mathbf{k}$, determine $\nabla\cdot(\nabla \times \mathbf{F})$. Use words to say what it is that is being computed.

(7) Find the first- and second-order Taylor polynomials for $f(x, y, z) = ye^{3x} + ze^{2y}$ at $\mathbf{a} = (0, 0, 2)$.

(8) Identify and determine the nature of the critical points of $f(x, y) = e^{-y}(x^2 - y^2)$.

(9) A cylindrical metal can is to be manufactured from a fixed amount of sheet metal. Use the method of Lagrange multipliers to determine the ratio between the dimensions of the can with the largest capacity. **Please:** Set up the system of equations to solve; you need not solve it.