MULTIVARIABLE CALCULUS
EXAM 2
FALL 2018

Name:
Honor Code Statement and Signature:

Directions: Complete all problems. Justify all answers/solutions. Calculators/notes/texts/cellphones are not permitted – the only permitted item is a writing utensil. Best of luck.

(1) (a) Write the definition of the *directional derivative of* f *at* $\mathbf{a}$ *in the direction of* $\mathbf{u}$, $D_{\mathbf{u}}f(\mathbf{a})$.

(b) **Fill-in-the-blank** Theorem 6.2 (of Section 2.6) established that for differentiable f we have that $D_{\mathbf{u}}f(\mathbf{a})$ exists and can be computed as the ________________________________ of the gradient of f at $\mathbf{a}$ and $\mathbf{u}$.

(c) From a theorem of chapter 1, $D_{\mathbf{u}}f(\mathbf{a})$ can also be computed as $\|\nabla f(\mathbf{a})\|\|\mathbf{u}\|\cos(\Theta)$, where Θ is the measure of the angle between these two vectors. In terms of Θ, characterize when the directional derivative is maximized, minimized and zero.

Date: November 15, 2018.

(2) **Compute the path of steepest descent** Let $f(x, y) = xy$. Some level curves of f are sketched below. Find the curve traced by the path of steepest *descent* that goes through the point $(x, y) = (1, 2)$.

The figure on the following page comes from a Maple worksheet and gives a set of contour curves (with the values indicated in the command line) for the function $f(x, y) = xy$. Sketch the curve traced and indicate the direction of the path.

(3) Consider the following path,

$$\mathbf{x}(t) = (\cos(3t), \sin(3t), 2t^{3/2}).$$

(a) Find the velocity.

(b) Find the acceleration.

(c) Find the speed.

(d) Give the unit tangent vector when $t = \pi$.

(e) Find the length of the path between $t = 0$ and $t = 2$.

(4) Consider the vector field $\mathbf{F} = (y^3, y)$, which is sketched below. Consider the flow line through the point $(0, 5)$ (denote it $\mathbf{x}_1(t)$) and the flow line through the point $(0, -5)$ (denote it $\mathbf{x}_2(t)$).

(a) There is a qualitative difference between these two flow lines. What is it?

(b) What is the divergence of $\mathbf{F}$ at the point $(0, -5)$?

(c) Does the divergence of $\mathbf{F}$ along the path $\mathbf{x}_2(t)$ increase, decrease or remain constant as t grows? Justify.

(5) Determine the first-order **and** second-order Taylor polynomial of $f(x, y) = e^{2x}\cos(y)$ at the point $(0, \pi/2)$.

(6) Find the (absolute) maximum and minimum values of $f(x, y) = \sin(x)\cos(y)$ on the square $R = \{(x, y)|0 \leq x \leq 2\pi, 0 \leq y \leq 2\pi\}$.

(7) Let the dimensions of a box (i.e. a parallelepiped) be x inches, by y inches, by z inches. Use the method of Lagrange multipliers to determine the dimensions of the largest box if the sum of the length and the girth (i.e. the perimeter of the cross section perpendicular to the length axis - i.e. think of your waist size) can be at most 108 inches.