MULTIVARIABLE CALCULUS
EXAM 2
FALL 2013

Name:
Honor Code Statement:

Directions: Complete all problems. Justify all answers/solutions. Each problem is worth 10 points. Calculators/notes/texts/cell-phones are not permitted – the only permitted item is a writing utensil. Best of luck.

(1) Calculate the velocity, speed, and acceleration of the path $\mathbf{x}(t) = (t, t^2, t^3)$. Also, sketch an image of the path, using arrows to indicate the direction in which the parameter increases. (If it is difficult to sketch the path, consider sketching its projections onto the 3 coordinate planes.)

Date: November 14, 2013.

(2) Find the arc length parameter $s = s(t)$ for the path

$$\mathbf{x}(t) = e^{at} \cos bt \ \mathbf{i} + e^{at} \sin bt \ \mathbf{j} + e^{at} \ \mathbf{k}.$$

Also, express the original parameter t in terms of s and, thereby, reparametrize $\mathbf{x}$ in terms of s.

(3) Calculate the flow line $\mathbf{x}(t)$ of the vector field $\mathbf{F}(x, y, z) = 2\mathbf{i} - 3y\mathbf{j} + z^3\mathbf{k}$ at the point $\mathbf{x}(0) = (3, 5, 7)$.

(4) Establish the given identity.

$$\nabla \times (f\mathbf{F}) = f\ \nabla \times \mathbf{F} + \nabla f \times \mathbf{F}$$

(5) Find the first- and second-order Taylor polynomials for the given function f at the point $\mathbf{a} = (0, 0, 0)$.

$$f(x, y, z) = \frac{1}{x^2 + y^2 + z^2 + 1}$$

(6) Identify and determine the nature of the critical points of the following function,

$$f(x, y) = e^{-y}(x^2 - y^2).$$

(7) Find the points on the ellipse $3x^2 - 4xy + 3y^2 = 50$ that are nearest and farthest from the origin. (Hint: one can avoid use of the second derivative test for constrained local extrema. How? Regardless, I've attached a photocopy of page 288 of the text.)