MULTIVARIABLE CALCULUS
EXAM 2
## FALL 2013

Name:
Honor Code Statement:

Directions: Complete all problems. Justify all answers/solutions. Definitions are worth 4 points, true/false questions are worth 2 points each, and any other problem is worth 10 points. Calculators are not permitted. Best of luck.

(1) [5 points] Find the value of the directional derivative of $f(x, y, z) = 3x^2y^2z^2 + 2xyz + z$ at $\mathbf{a} = (4, 5, 6)$ in the direction of the vector $(1, 1, 2)$.

(2) [5 points] Calculate the velocity, speed, and acceleration of the following path: $\mathbf{x}(t) = (5 \sin t, 3 \cos t)$.

(3) [5 points] Calculate the length of the path $\mathbf{x}(t) = (t^3, 3t^2, 6t)$ for $-1 \leq t \leq 2$.

(4) [5 points] The following figure is the vector field for the function $\mathbf{F}(x, y) = (y, -x)$. Use the figure to sketch the flow line that contains the point $(2, 0)$.
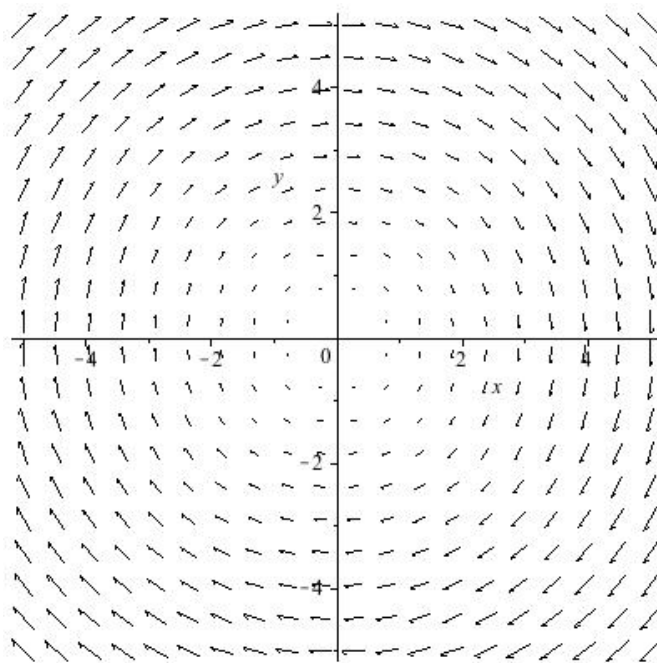
(5) [5 points] Is it possible for the vector field of the previous problem to also be a gradient field? If yes, then find a scalar-valued function f that would produce this $\mathbf{F}$. If no, explain why.

(6) [5 points] Why would asking you to find the curl of $\mathbf{F}$, where $\mathbf{F}$ is as given in problem 4, be a trick question?

(7) [10 points] Find the first- and second-order Taylor polynomials for the function $f(x, y) = xy + \sin x \cos y$ at $\mathbf{a} = (\pi, \pi)$.

(8) [10 points] Identify and determine the nature of the two critical points of the function $f(x, y, z) = x^3 + xz^2 - 3x^2 + y^2 + 2z^2$.

(9) [10 points] A farmer has determined that her cornfield will yield corn (in bushels) according to the formula

$$B(x, y) = 4x^2 + y^2 + 600,$$

where x denotes the amount of water (measured in hundreds of gallons) used to irrigate the field and y the number of pounds of fertilizer applied to the field. The fertilizer costs \$10 per pound and water costs \$15 per hundred gallons (wow, expensive water!). If she can allot \$500 to prepare her field through irrigation and fertilization, use a Lagrange multiplier to determine how much water and fertilizer she should purchase in order to maximize her yield.

(10) [5 points] Let $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$. Verify that $\nabla||\mathbf{r}||^n = n||\mathbf{r}||^{n-2}\mathbf{r}$.