MULTIVARIABLE CALCULUS
EXAM 2
FALL 2012

Name:
Honor Code Statement:

Directions: Complete all problems. Justify all answers/solutions. Definitions are worth 4 points, true/false questions are worth 2 points each, and any other problem is worth 10 points. Calculators are not permitted. Best of luck.

(1) Calculate the directional derivative of \( f(x, y, z) = xyz \) at the point \( a = (-1, 0, 2) \) in the direction parallel to the vector \( \mathbf{u} = \frac{2k - i}{\sqrt{5}} \).

(2) Define: curvature \( \kappa \) of a path \( \mathbf{x} \) in \( \mathbb{R}^3 \).
(3) Calculate the plane tangent to the surface whose equation is $x^2 - 2y^2 + 5xz = 7$ at the point $(-1, 0, -\frac{5}{2})$ by first computing the gradient (and then using this computation).

(4) Compute the length of the path $\mathbf{x} : [0, 2\pi] \to \mathbb{R}^2$, $\mathbf{x}(t) = (3\cos(t), 3\sin(t))$. 
(5) Verify that the path \( \mathbf{x}(t) = (\sin t, \cos t, 2t) \) is a flow line of the vector field \( \mathbf{F} = (y, -x, 2) \).

(6) Find the first- and second-order Taylor polynomials for the function \( f(x, y) = e^{2x} \cos(3y) \) at \( \mathbf{a} = (0, \pi) \).
(7) Identify and determine the nature of the critical points of the function $f(x, y) = x^2 - y^3 - x^2 y + y$. 
(8) Set up (but do not solve) the system of equations (via the method of Lagrange multipliers) for finding the critical points of \( f(x, y, z) = x + y + z \) subject to the constraints \( x + 2z = 1 \), \( y^2 - x^2 = 1 \).
(9) **True or False:** Mark true statements as True, false statements as False. Give a brief justification or correct the statement (in as minimal a way as possible) so as to make it true.

(a) If a path is parametrized by arc length, then its velocity and acceleration are orthogonal.

(b) \( \text{grad} \: f \) is a scalar field.

(c) \( \text{div} \: F \) is a vector field.

(d) If a path \( x \) remains a constant distance from the origin, then the velocity of \( x \) is perpendicular to \( x \).

(e) \( \nabla \times (\nabla f) = 0 \) for all functions \( f : \mathbb{R}^3 \to \mathbb{R} \).

(f) If \( f \) is differentiable and has a local extremum at \( a = (a_1, \ldots, a_n) \),
then \( \nabla f(a) = 0 \).

(g) The set \( \{ (x, y, z) | 4 \leq x^2 + y^2 + z^2 \leq 9 \} \) is compact.

(h) The increment \( \Delta f \) of a function \( f(x, y) \) measures the change in the z-coordinate of the tangent plane to the graph of \( f \).