MULTIVARIABLE CALCULUS
EXAM 1
SPRING 2021

Name:
Honor Code Statement:

Directions: You may OMIT one five point question; indicate which you omit by putting a slash through it. Justify all answers/solutions. Calculators, notes, texts and collaboration are not permitted. A formula sheet is on the last page. Best of luck!

(1) [5 points] Write a set of parametric equations for the line in $\mathbb{R}^4$ through the points $(1, 1, 1, 1)$ and $(1, 2, 3, 4)$.

Date: March 25, 2021.

(2) [5 points] Find three vectors that are perpendicular to $\mathbf{i} - \mathbf{j} + \mathbf{k}$ such that no two of these are parallel.

(3) [5 points] Find the volume of the parallelepiped determined by

$$\mathbf{a} = 3\mathbf{i} - \mathbf{j}, \ \mathbf{b} = -2\mathbf{i} + \mathbf{k}, \ \mathbf{c} = \mathbf{i} - 2\mathbf{j} + 4\mathbf{k}.$$

(4) [5 points] Calculate the distance between the two planes:

$$3x - 3y + 2z = 8, \; -12x + 12y - 8z = 0.$$

(5) [5 points] Let's try a short proof. Let $\mathbf{a}, \mathbf{b}$ be vectors in $\mathbb{R}^n$. Let's assume that $||\mathbf{a} + \mathbf{b}|| = ||\mathbf{a} - \mathbf{b}||$. You will prove that $\mathbf{a}$ and $\mathbf{b}$ are orthogonal by squaring both sides of the given equation, "multiplying out", and collecting like terms. Do this.

(6) [5 points] Draw a picture that illustrates the situation in the previous problem for when $\mathbf{a}, \mathbf{b}$ are vectors in $\mathbb{R}^2$. (If you haven't yet given the above proof, you can still attempt this problem.)

(7) [10 points] Given the function $f(x, y) = \sqrt{x^2 + y^2}$ sketch 3 level curves. Sketch **or** describe the surface.

(8) [5 points] Show that the following limit does not exist.

$$\lim_{(x,y)\to(2,0)} \frac{x^2 - y^2 - 4x + 4}{x^2 + y^2 - 4x + 4}.$$

(9) [10 points] Give the equation of the tangent plane of the given function at the given point.

$$f(x, y) = ((x-1)y)^{2/3},\ (a, b) = (1, 0)$$

(10) [10 points] Consider the following function at the following point.

$$f(x, y) = e^y \sin x, \mathbf{a} = (\frac{\pi}{3}, 0)$$

(a) Compute the direction of steepest descent.

(b) Compute a direction for maintaining the current "altitude".

(c) Compute the directional derivative in the direction parallel to the vector $\mathbf{v} = \mathbf{i} - \mathbf{j}$.

Change of coordinates

Cylindrical to Cartesian:

$$x = r\cos\theta,\ y = r\sin\theta, z = z$$

Cartesian to cylindrical:

$$r^2 = x^2 + y^2,\ \tan(\theta) = \frac{y}{x},\ z = z$$

Spherical to Cartesian:

$$x = \rho\sin\varphi\cos\theta,\ y = \rho\sin\varphi\sin\theta,\ z = \rho\cos\varphi$$

Cartesian to spherical:

$$\rho^2 = x^2 + y^2 + z^2,\ \tan(\varphi) = \sqrt{x^2+y^2}/z,\ \tan(\theta) = \frac{y}{x}$$

Spherical to cylindrical:

$$r = \rho\sin(\varphi),\ \theta = \theta,\ z = \rho\cos(\varphi)$$

Cylindrical to spherical:

$$\rho^2 = r^2 + z^2,\ \tan(\varphi) = r/z,\ \theta = \theta$$