MULTIVARIABLE CALCULUS
EXAM 1
SPRING 2018

Name:
Honor Code Statement:

Directions: Complete all problems. Justify all answers/solutions. Calculators, notes, texts and collaboration are not permitted. Best of luck.

(1) [10 points] Give an equation for the plane parallel to the plane $2x-3y+z=5$ that passes through the point $(-1,1,2)$. **Also,** find the distance between the given plane and point.

Date: March 15, 2018.

(2) [10 points] **Graph paper is available, which should improve your accuracy and conclusions you make.** Determine and draw several – say, at least four – level curves of the given function f (and make sure to indicate the height c of each curve).

$$f(x, y) = \sqrt{x^2 + y^2}$$

Now give and draw four sections of the graph of f by planes of the form $y = c$, where c is a constant.

Use the above information to sketch the graph of f **or** describe it accurately in words.

(3) [5 points] Determine the following limit, if it exists. Hint: make a change of variable.

$$\lim_{(x,y)\to(0,0)} \frac{x^2 + xy + y^2}{x^2 + y^2}$$

(4) [5 points] State what it means for a function to be **continuous**.

(5) [3 points] Choose the correct term to describe the following set. $S := \{(x, y, z) \in \mathbb{R}^3 | 1 < x^2 + y^2 < 4\}$. Terms: open, closed or neither. Justify your answer.

(6) [3 points] True or false? There exists two vectors in S whose cross product is also in S. Justify your answer.

(7) [4 points] Do there exist $\mathbf{p}, \mathbf{q}, \mathbf{r} \in \mathbb{R}^2$ with $||p - q|| = ||q - r|| = 1$ and $||p - r|| = 3$?

(8) [10 points] Consider the graph $z = x^3 - 7xy + e^y$. Find the tangent plane at $(-1, 0, 0)$. Is this tangent plane a good linear approximation to the graph near the point of tangency? Why?

(9) [5 points] *Use the product rule* to find the derivative of the product of the following two functions: $f(x, y, z) = e^{x+y}$ and $g(x, y, z) = xy + xz + yz$.

(10) [10 points] Suppose that a bird flies along the helical curve $x = 2\cos t, y = 2\sin t, z = 3t$. The bird suddenly encounters a weather front so that the barometric pressure is varying rather wildly from point to point as

$$P(x, y, z) = \frac{6x^2 z}{y} atm.$$

Use the chain rule to determine how the pressure is changing at $t = \frac{\pi}{4}$.

Change of coordinates

Cylindrical to Cartesian:

$$x = r\cos\theta,\ y = r\sin\theta, z = z$$

Cartesian to cylindrical:

$$r^2 = x^2 + y^2,\ \tan(\theta) = \frac{y}{x},\ z = z$$

Spherical to Cartesian:

$$x = \rho\sin\varphi\cos\theta,\ y = \rho\sin\varphi\sin\theta,\ z = \rho\cos\varphi$$

Cartesian to spherical:

$$\rho^2 = x^2 + y^2 + z^2,\ \tan(\varphi) = \sqrt{x^2 + y^2}/z,\ \tan(\theta) = \frac{y}{x}$$

Spherical to cylindrical:

$$r = \rho\sin(\varphi),\ \theta = \theta,\ z = \rho\cos(\varphi)$$

Cylindrical to spherical:

$$\rho^2 = r^2 + z^2,\ \tan(\varphi) = r/z,\ \theta = \theta$$