MULTIVARIABLE CALCULUS
EXAM 1
FALL 2019

Name:
Honor Code Statement:

Directions: Complete all problems. Justify all answers/solutions. Calculators, notes, texts and collaboration are not permitted. A formula sheet is on the last page. Best of luck.

(1) [5 points] Find three nonparallel vectors that are perpendicular to $\mathbf{i}-\mathbf{j}+\mathbf{k}$.

(2) [5 points] Use the cross product to find the area of the triangle determined by the vectors $\mathbf{a}=(1,-2,6)$ and $\mathbf{b}=(4,3,-1)$.

Date: October 10, 2019.

(3) [5 points] Calculate the distance between the two parallel planes

$$5x - 2y + 2z = 12$$

and

$$-10x + 4y - 4z = 8$$

.

(4) [2 points] In one sentence, explain how one can tell these two given equations yield *parallel* planes.

(5) [6 points] The following expressions are “nonsense”, that is, these are undefined. Make the smallest number of changes possible to each expression, restricting yourself to the insertion, deletion or replacement of a binary operation, so that the expression now has meaning. Indicate whether your new expression is a scalar, a vector or something else.

- $(\mathbf{a} \cdot \mathbf{b}) \cdot \mathbf{c}$
- $(\mathbf{a} \cdot \mathbf{b}) \times (\mathbf{c} \cdot \mathbf{d})$
- $(\mathbf{a} \cdot \mathbf{b}) \times (\mathbf{c} \times \mathbf{d})$

(6) [4 points] Write what it means to say for a scalar-valued function f to be **differentiable** at $\mathbf{a} = (a_1, \ldots, a_n)$.

(7) [10 points] Consider the surface in $\mathbb{R}^3$ determined by the following equation, $z = y^2 + 2$.. (Yes, you read it correctly. This is a surface in $\mathbb{R}^3$ and x does not appear in the equation.) Sketch 3 level curves. Sketch two sections of the graph of f by the plane $x = c$. Sketch two sections of the graph of f by the plane $y = c$. Sketch **or** describe the surface.

(8) [6 points] Calculate the following limit by first change from Cartesian coordinates to polar coordinates.

$$\lim_{(x,y)\to(0,0)} \frac{x^2 - y^2}{\sqrt{x^2 + y^2}}$$

(9) [6 points] Find an equation for the hyperplane tangent to the 4-dimensional paraboloid

$$x_5 = 10 - (x_1^2 + 3x_2^2 + 2x_3^2 + x_4^2)$$

at the point $(2, -1, 1, 3, -8)$.

(10) [6 points] Fill in the blank. Suppose that two surfaces are given by the equations $F(x, y, z) = c$ and $G(x, y, z) = k$. Further, suppose that the surfaces intersect at the point (x_0, y_0, z_0). The surfaces are tangent at (x_0, y_0, z_0) if and only if the tangent planes at (x_0, y_0, z_0) are the ____________________ if and only if the ____________________ vectors at (x_0, y_0, z_0) are parallel if and only if $\nabla F(x_0, y_0, z_0) \times \nabla G(x_0, y_0, z_0)$ equals ____________________.

(11) [5 points] Find the directional derivative of $f(x, y, z) = \frac{xe^y}{3z^2+1}$ at the point $\mathbf{a} = (2, -1, 0)$ in the direction parallel to the vector $\mathbf{u} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$. Is there a direction for which the directional derivative is larger?

(12) [5 points] The following page shows a “proof” of the Cauchy-Schwarz Inequality. One of the inequalities is circled. Explain why this circled inequality is true.**3 sentences maximum**

Change of coordinates

Cylindrical to Cartesian:

$$x = r\cos\theta,\ y = r\sin\theta, z = z$$

Cartesian to cylindrical:

$$r^2 = x^2 + y^2,\ \tan(\theta) = \frac{y}{x},\ z = z$$

Spherical to Cartesian:

$$x = \rho\sin\varphi\cos\theta,\ y = \rho\sin\varphi\sin\theta,\ z = \rho\cos\varphi$$

Cartesian to spherical:

$$\rho^2 = x^2 + y^2 + z^2,\ \tan(\varphi) = \sqrt{x^2+y^2}/z,\ \tan(\theta) = \frac{y}{x}$$

Spherical to cylindrical:

$$r = \rho\sin(\varphi),\ \theta = \theta,\ z = \rho\cos(\varphi)$$

Cylindrical to spherical:

$$\rho^2 = r^2 + z^2,\ \tan(\varphi) = r/z,\ \theta = \theta$$