MULTIVARIABLE CALCULUS
EXAM 1
FALL 2014

Name:
Honor Code Statement:

Directions: Complete all problems. Justify all answers/solutions. Calculators, notes, texts and collaboration are not permitted. Best of luck.

(1) [5 points] Give a set of parametric equations for the line through $(5, 0, 6)$ that is perpendicular to the plane $2x - 3y + 5z = -1$.

Date: October 9, 2014.

(2) [5 points] Find a value for A so that the planes $8x - 6y + 9Az = 6$ and $Ax + y + 2z = 3$ are parallel.

(3) [5 points] Find the distance between the two planes from the previous problem.

(4) [5 points] Briefly explain why the following set is not open: $X = \{(x, y) : 1 \leq x \leq 2, 1 \leq y < 3\}$.

(5) [5 points] State what it means for a scalar-valued function to be *smooth*.

(6) [5 points] A function $f(x, y)$ has $f(1, 2) = 3$ and $\lim_{(x,y)\to(1,2)} f(x, y) = 4$. Does this tell you anything about the differentiability at $(1, 2)$? What? Justify your answer.

(7) [10 points] Determine (and draw) several – say, at least four – level curves of the given function f (and make sure to indicate the height c of each curve).

$$f(x, y) = |x|$$

Now give (and draw) four sections of the graph of f by planes of the form $y = c$, where c is a constant.

Use the above information to sketch the graph of f or describe it accurately in words.

(8) Find an equation for the hyperplane tangent to the 4-dimensional paraboloid $x_5 = 10 - (x_1^2 + 3x_2^2 + 2x_3^2 + x_4^2)$ at the point $(2, -1, 1, 3, -8)$.

(9) [5 points] Let $\mathbf{g} : \mathbb{R}^3 \to \mathbb{R}^2$ be a differentiable functions such that $\mathbf{g}(1, -1, 3) = (2, 5)$ and $D\mathbf{g}(1, -1, 3) = \left[\begin{array}{ccc} 1 & -1 & 0 \\ 4 & 0 & 7 \end{array} \right]$ Suppose that $\mathbf{f} : \mathbb{R}^2 \to \mathbb{R}^2$ is denied by $\mathbf{f}(x, y) = (2xy, 3x - y + 5)$. What is $D(\mathbf{f} \circ \mathbf{g})(1, -1, 3)$?

(10) Consider a surface with height $f(x, y) = 10 - x^2 - y^2$,

- Find the direction to head if starting at the point $(1, 2, 5)$ if you wish to experience steepest descent.

- Now find the path of steepest descent starting from this point.

(11) [5 points] Use the Cauchy-Schwarz inequality to prove that for real numbers $d_1, \ldots, d_n$ we always have

$$d_1^2 + \ldots + d_n^2 \geq \frac{(d_1 + \ldots + d_n)^2}{n}$$

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