MULTIVARIABLE CALCULUS
EXAM 1
FALL 2013

Name:
Honor Code Statement:

Directions: Complete all problems. Justify all answers/solutions. Calculators, notes, texts and collaboration are not permitted. Best of luck.

(1) [5 points] Find the angle between the following two non-zero vectors: $2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$ and $\mathbf{i} + \mathbf{k}$.

(2) [5 points] State the Triangle Inequality. Indicate under what conditions equality holds.

Date: October 10, 2013.

(3) [10 points] Give an equation for the plane containing the following three non-collinear points $(1,0,0),(0,5,0),(0,0,2)$.

Now give the set of parametric equations for this plane.

(4) [10 points] Determine (and draw) several – say, at least four – level curves of the given function f (and make sure to indicate the height c of each curve).

$$f(x, y) = 4x^2 + 9y^2$$

Now give (and draw) four sections of the graph of f by planes of the form $x = c$, where c is a constant.

Use the above information to sketch the graph of f.

(5) [5 points] Use a change of coordinates to polar coordinates to find the following limit. (Recall that $x = r\cos(\theta)$, $y = r\sin(\theta)$ and $r^2 = x^2 + y^2$ and $\tan(\theta) = \frac{y}{x}$ are the polar to cartesian and cartesian to polar change of coordinates, respectively.) **Hint:** $-1 \leq \cos(\theta) \leq 1$.

$$\lim_{(x,y)\to(0,0)} \frac{x^3 + x^5}{x^2 + y^2}$$

(6) [10 points] Find an equation for the plane tangent to the graph of $f(x, y) = 4\cos(xy)$ at the point $\mathbf{a} = (\pi/3, 1, 2)$.

This tangent plane is a good linear approximation of f near $\mathbf{a}$ only if f is differentiable at $\mathbf{a}$. Since the existence of the above plane doesn't guarantee differentiability, how can you be sure that f is differentiable at $\mathbf{a}$?

(7) [5 points] Calculate $D(f \circ \mathbf{g})$ in two ways: (a) by first evaluating $f \circ \mathbf{g}$ and (b) by using the chain rule and the derivative matrices Df and $D\mathbf{g}$.

$$f(x, y) = x^2 - 3y^2, \ \mathbf{g}(s, t) = (st, s + t^2)$$

(8) [5 points] Suppose that the four vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$, and $\mathbf{d}$ in $\mathbb{R}^3$ are coplanar (i.e., they all lie in the same plane). Show that then $(\mathbf{a} \times \mathbf{b}) \times (\mathbf{c} \times \mathbf{d}) = \mathbf{0}$.