FYSE 1314 — THE MATHEMATICAL GARDNER

ASSIGNMENTS

All readings, unless otherwise noted, come from *Martin Gardner's Mathematical Games*. Please remember that reading a chapter (an article) means reading **AND** puzzling, i.e building objects, performing tricks, solving puzzles, etc. Responses to questions should be typed and brought to class; always assume double-spacing. It may be helpful for you to discuss some problems posed with your classmates. You may do so, however the *write up of the solutions must be done on your own.*

(1) Due date: Thursday, September 15
Read: Gardner10, Chapter 7, *The Combinatorics of Paper Folding.*
As a result, you will have created a set of stamp books of various sizes, created the dictators puzzle (and solved it), created the Beelzebub puzzle, a tetraflexagon, and the sheep and goats puzzle.

Write: On page 61 there are eight permutations that can not be folded with a strip of stamps of length four. Pick one of these eight and explain why it cannot be folded (2 page limit). And to be discussed in class: can you find the recursive procedure alluded to on the top of page 61 and describe it?

(2) Due date: Tuesday, September 20
Read: Gardner 4, Chapter 8, *A Matchbox Game-Learning Machine.*
Build the learning machine for hexapawn, HER (Hexapawn Educable Robot). Once you have built HER, play against HER as indicated in the article to turn HER into a perfect-gamer. Should we have a class tournament when you bring your robot to class? (Your robot might play against me in an exam.)

Write: Please create two questions, and describe any progress you've made on them.

(3) Due date: Thursday, September 22
Read: Gardner 1, Chapter 8, *The Game of Hex.*
Be prepared to play in class. (Proof of first-player win makes a nice exam question.) The theorem that the game Hex cannot result in a draw seems fairly obvious. However, it is equivalent to a rather famous theorem in topology known as the Brouwer Fixed-Point Theorem. Please check out of the library a topology text that includes a copy of the Brouwer Fixed-Point Theorem; read the theorem and bring the text to class.

Consider the Hex board with seven hexagons per side. Let white pieces occupy cells C3 and F5 and a black piece occupy B6. Black is to play and can guarantee a win on his next move. Determine this move and White's best defense.

(4) Due date: Tuesday, September 27
Read: Gardner 6, Chapter 1, *Sprouts and Brussel Sprouts.*
Be prepared to play sprouts in class. Note the reference on page 4 to the Jordan-curve theorem. The topology text that you checked out of the library should contain the statement of this theorem. What is meant by a *simple closed curve*?

Write: Prove that each game of sprouts must last at least $2n$ moves. Prove that Brussels sprouts must end in $5n - 2$ moves, and then deduce the corollary about first-player win if one starts with an odd number of crosses, and loses otherwise.

(5) Due date: Thursday, September 29
Read: Gardner 3, Chapter 18, *Bridg-it and Other Games* and Gardner 6, Chapter 16, *Jam, Hot and Other Games.*
Be prepared to play hamstrung squad car. So, have a board ready to play on. Begin analysis of the game to determine the fatal area.

Write: Create an unsafe position (consisting of at least four piles, and ten counters) in the game of nim, and determine a move that returns it to a safe position. (Determining when a nim position is safe or not makes a nice exam question.)

(6) Due date: Thursday, October 6
Read: Gardner 10, Chapter 14, *Nim and Hackenbush* and Gardner 5, Chapter 20, Coleridge's Apples and Eight Other Problems, Problem 2
Be prepared to play hackenbush. Also, assigned groups of three will perform the topological trick of reversing a person's trousers - prizes to be awarded for fastest and most entertaining.

(7) Due date: Tuesday, October 11
Read: Gardner 3, Chapter 1, *The Binary System*
Create your own set of punch-cards. Use these to solve each of the problems discussed in the reading.

Assigned groups of three will create their own puzzle or logic problem to be solved. Each of the five problems will be presented in class for the other groups to solve. (What might you want to achieve in creating your problem?)

Write: each person will write about the puzzle/problem created by the group and give its solution.

(8) Due date: Thursday, October 13
Read Gardner02, Chapter 15, *Eleusis: The Induction Game*

As a result, you will have a deck of cards, be familiar with the rules and scoring, and be ready to play the game in class.

Write: Please have prepared three "rules" for the times when you are dealer, written with the intention of winning (and of course, clarity is always important). Each should be written on a separate sheet of paper, leaving space to record the scoring of the hand for a given rule. These will be turned in and your score during play will be incorporated into your score for the assignment.

Related reading, which you may read if you wish: Gardner13, Chapter 16, *The New Eleusis* and Gardner08, Chapter 4, *Patterns of Induction*

(9) Due date: Thursday, October 20

Write: A 2-page description of the topic you will discuss during "your" ten minutes of the 20-minute oral exam. The description should contain some ideas, questions, approaches, propositions, or theorems of others and your own ones as well. You will have practiced your presentation in front of two other group members (groups assigned in class) and please write about the feedback you received from them as well. (This and all future writing assignments that I collect will receive a grade.)

For your ten minutes of the oral exam: You may bring any materials with you. The ten minutes will be the first ten minutes of our meeting time and I will cut you off if you run over. The exam will occur in my office, where there is a small chalkboard. (If this is not sufficient, please let me know what space you might need.)

For the second ten minutes of the oral exam: You should bring your box of mathematical curiosities, and be prepared to answer questions from the readings.

I will bring a sign-up sheet for scheduling these exams. I've set aside Friday, the 21st of October, 9-11am and 1:30-4:30pm, and then Monday the 24th, 9-11am and 1:30-2:30pm.

(10) Due date: Tuesday, October 25

We are scheduled to dine in the home of the Ross Commons faculty heads, Maria Hatjigeorgiou and Pavlos Sfyroeras, on Tuesday evening at 6pm. Their home is at 26 Blinn Lane. Please think about how we might show our appreciation for their hospitality.

Read: Gardner13, Chapter 5, *Back from the Klondike and Other Problems.*

Most interesting to me are the pawn problems (which are really queen problems, if one thinks about chess movements) at the end of this article and I'd like for us to explore them for an extended period of time.

Our Tuesday class meeting will be a discussion led by me. (How unusual!)

Consider the minimum no-3-in-a-line problem with the "narrow" definition of line. For board sizes, say, 10, 20 and 40, can you devise a placement meeting the requirements that uses very few queens? Give these and write about how you devised these placements. (If you have an arithmetic or geometric reason to choose different board sizes, then go ahead. Just make them 'small, medium, and large'.)

Be prepared to play the game mentioned on page 71. (Implementing Knuth's rule to turn a magic permutation into a maximal placement makes a good exam question.)

(11) Due date: Thursday, October 27

Please be prepared to engage in a discussion that addresses the following questions concerning our class.

- Is there a point to what we are doing? If so, what?
- Why play?
- Is what we're doing valuable? If yes, how so? If no, why not? If what we are doing is valuable, are we executing well? What could we be doing better?
- Are you learning something new?
- Does this course fit into a liberal arts education?
- Is the course falling short of your expectations as a whole or with respect to some particular aspect, such as writing?

Write: Please sketch your responses for each of these questions.

Sean will moderate the discussion (and so he will decide on the order we consider the questions, and how long we spend with each, while ensuring that each person contributes to the conversation).

(12) Due date: Tuesday, November 1

Please see my e-mail to you.

(13) Due date: Tuesday, November 8

Read: Gardner10, Chapter 20, *The Game of Life, Part I*, Chapter 21 *The Game of Life, Part II*, and Chapter 22 *The Game of Life, Part III.*

Visit www.conwaylife.com for faster play of some larger initial configurations and read more about the game and its devotees. You will find numerous other websites for play; you needn't stick with the above one.

Devise activities or discussion questions for Tuesday's meeting (and beyond?) based on these readings. (Would you be able to "play" one step of the game by hand?)

As registration for spring semester is upcoming, please visit me in my office over the next week to discuss course planning. Please try the "practice registration" via Bannerweb prior to visiting me. If you do not visit me, you will be unable to register for your courses.

(14) Due date: Thursday, November 10

Prepare for debate on the following question: does free will exist?

(15) Due date: Tuesday, November 15

Read: The Card Game Set, by B. Davis and D. Maclagan. A copy of their manuscript can be found here, https://homepages.warwick.ac.uk/staff/D.Maclagan/papers/set.pdf .

Write: Figure 11 of the article: how much progress can you make on this puzzle? (The statement and proof of Proposition 1 makes a good exam question.) The article addresses finding the largest size of a maximal cap set. Note that a maximal cap set does not contain three cards forming a SET. Can you find a "small" maximal cap set? i.e., can you find a collection of cards not containing a SET such that when you add one more card there is guaranteed to be a SET regardless of the card that you add?

For class: be prepared to share progress on these problems and to play the game. Please bring the card game to class.

(16) Due date: Thursday, November 17

Read: https://www.quantamagazine.org/20160531-set-proof-stuns-mathematicians/ . Please also follow the links embedded in the article to familiarize yourself with the people involved with the progress on this problem and what has been written about it. (Those taking linear algebra will see a direct benefit of that class in what is being discussed in these readings.) A re-read of the Davis-Maclagan manuscript is also in order.

For class, we will: (1) continue a discussion of Proposition 1 as led by me (it seems that there are many good questions to ask on an oral exam), (2) have a member or members of the class present solution to puzzle of Figure 11, and (3) discuss findings presented in new article.

(17) Due date: Tuesday, November 19
Class will be held in my home at 1 South Gorham Lane. We'll be watching a documentary on Martin Gardner - popcorn to be served!

A sketch of the manuscript you are preparing should be made and sent to Josh Nislick (jnislick@middlebury.edu) by Tuesday at noon.

(18) Due date: Thursday, December 1

Read: Gardner 7, Chapter 3, *Game Theory, Guess It, Foxholes.*

For class: We will implement the "urging" given in the first full paragraph on page 42 of the reading. That is, we'll play Guess It in the three manners described, so please bring all necessary supplies.

Write: In the last sentence of the reading, Gardner quotes Isaacs, "with even n one encounters some modest novelty." So, can you take up this challenge?

(19) Due dates: Monday, December 5 and Tuesday, December 6

Write: Your writing projects are due to my office on ~~Monday, December 5th~~ Wednesday, December 7th by 5pm.

Write (to share in class): Please respond to the following questions. What have you learned from this course? How have your analytical abilities and creativity been deepened? How did the various components of the course contribute to your learning? What would you maintain about the course, its content, its methods, etc.? What would you change?

In class: We will devote 20-30 minutes of class-time to discussing these questions. Ronnie will moderate the discussion.

(20) Due date: Thursday, December 8

Read: Gardner 11, Chapter 7, *Napier's Bones*

Create: Make a set of Napier's bones. Create a multiplication problem that you are able to solve using Napier's bones and be prepared to present the solution to the class.

Write: There is a little combinatorics puzzle on the top of page 90: "can the reader determine the largest multiplicand one set of Napier's bones will form such that all smaller multiplicands can also be formed by the set?" What progress can you make on this problem? We will discuss as a class.

(21) Oral final exam is the week of December 12th. You will have signed up for a time-slot in class. The exam will be twenty minutes, with ten minutes reserved for you to present on some topic and the second ten minutes is reserved for me to question you on topics related to the readings. Please remember to bring your more recent bag of supplies (which will include Napier's bones, Guess It spinners, and more – I will have the SET game, in case you don't have the game).