“Bucket auctions” for charity

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ABSTRACT

Donations in-kind can be a mixed blessing for charities, who are often more adept at solicitation than resale. Many organizations rely on raffles to turn donations into cash, but auctions are also common. Theory predicts that all-pay mechanisms should produce more revenue than winner-pay mechanisms, but the empirical literature is thin and inconclusive. Drawing on both theoretical insights and behavioral intuition, we examine another all-pay mechanism, the “bucket auction,” and show that it generates more revenue than other standard mechanisms, both in theory and in the lab. We hope, therefore, that this format, and others like it, will attract the interest of fundraisers.

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1. Introduction

More than 80 percent of the almost 50 million itemized personal tax returns filed in the United States during 2008 claimed charitable donations and of these, 23 million listed non-cash donations.1 The reported value of these non-cash donations was about $40 billion, or about 20 percent of the total. There is some evidence that donations in-kind are even more important in a recession. The value of all charitable donations is estimated to have fallen 3.2% in 2009, but the value of business donations increased 5.5%, and almost all of this could be attributed to contributions in-kind, which rose almost 30% (Giving USA, 2010). These donations pose particular challenges for charities and non-profits, which often have more experience with the solicitation of gifts than their subsequent transformation into cash. For better or worse, the simple raffle remains a common, if sometimes perfunctory, choice of mechanism, but a small number of standard auction mechanisms (sealed bid, oral or ascending and silent) are now common, too. For example, Giving Works, the charity auction arm of eBay, raised $36 million over the course of one year (2008), and on a single evening in May 2008, the Robin Hood Foundation, which targets poverty in New York City, auctioned $57 million worth of prizes to a crowd of celebrities and hedge fund managers. Despite fundraisers’ increased reliance on auctions, however, it is not clear, based on the scant empirical literature, which of the common formats is the most effective, or whether new, even more lucrative, mechanisms couldn’t be designed.

From a theoretical perspective, the salient feature of a charity auction is the existence of revenue proportional benefits that accrue to all bidders, win or lose. As a consequence, revenue equivalence fails even in the benchmark case of
independent private values and mechanisms can be ordered in terms of expected revenue. For example, it is now well understood (Engelbrecht-Wiggans, 1994; Goeree et al., 2005; Engers and McManus, 2006) that second price sealed bid auctions should do better than first price auctions, but that both should do worse than the less familiar all-pay auction, in which participants forfeit their bids, win or lose. The intuition is that a bidder who tops the current high bid in a winner-pay auction stands to win the object but lose whatever benefits the topped bid would have produced. This disincentive to compete is absent in all-pay auctions, however, because the revenue proportional benefits associated with topped bids are not lost. Goeree et al. (2005) extend this result, and show that a second price all-pay mechanism, in which all bidders but the winner forfeit their bids, and the winner forfeits an amount equal to the second highest bid, should do even better than a first price all-pay mechanism. The explanation turns on the fact that losers have some incentive to “plump” their bids because of the effects on winners’ payments.

Much of the empirical evidence we have comes from experimental studies in the lab and offers support for the broad prediction that all-pay mechanisms, understood here to include raffles and lotteries, do better than winner-pay mechanisms.2 For example, although Schram and Onderstal (2009) and Corazzini et al. (2010) reach different conclusions about which is the most effective all-pay format, the former does find that the first price all-pay and raffle outperform the first price winner-pay. In addition, Davis et al. (2006) find that the raffle does better than the oral ascending (a.k.a., English) auction, another familiar winner-pay mechanism. Last, consistent with Goeree et al. (2005), Orzen (2008) concludes that the last price all-pay, in which the highest bidder wins but all bidders pay an amount equal to the lowest bid, surpasses the first price all-pay.3

Inspired by the possibilities for revenue enhancement in the theoretical literature and by the strong performance of all-pay mechanisms in the lab, we describe another deceptively simple format, which we call the “bucket auction.” Potential bidders are asked to form a circle, one of whom is then selected, at random, to start the auction. That individual is given a “bucket” and must decide whether to drop a non-refundable “chip” — that is, a fixed and non-refundable increment — into it, or withdraw from the circle/auction. The bucket is then passed on to the next bidder, who must make the same decision. It then travels around the circle until the prize is awarded to the last bidder who contributes a chip.

As our model in the third section confirms, there was good reason to believe that the bucket auction would do well, quite apart from the behavioral factors that we show also contribute to its success. While our model allows bidders to condition their behavior on the number of competitors still in the circle — that is, we consider closed loop strategies — the intuition is similar to the open loop case, when the bucket, which resembles a war of attrition, is equivalent to a second price all-pay auction (Krishna and Morgan, 1997; Dekel et al., 2006). In this context, the logic of Goeree et al. (2005) is applicable: “bid toppers” don’t forfeit the positive externalities associated with their rivals’ bids and also benefit from “bid plumping.”

Although unfamiliar to us when we first described the bucket auction – indeed, there is some chance it didn’t exist, at least in its current form – our mechanism bears some resemblance to the “penny auction,” a modified ascending auction in which bidders are welcome to top the current high bid by some small fixed increment (usually $0.01, which explains the name) but must submit some substantial and non-refundable fee for each bid. After examining data from more than 150,000 online penny auctions on Swoopo, until 2011 the largest such auctioneer, Augenblick (2012) finds that mean revenues exceed 150% of the good’s (maximum) private value. There are at least two important differences between the bucket mechanism and these auctions, however. The first is that under the rules of the bucket, all bidders must bear continuation costs, and in this sense is similar to the standard “war of attrition,” a feature we exploit when we derive theoretical predictions. The second, of course, is that under our charity-oriented rules, the bucket mechanism also generates substantial positive externalities, in the form of revenue proportional benefits to all bidders. This said, we believe that an experimental evaluation of the bucket in the absence of such benefits – that is, in a for-profit environment – would be an important extension of this research, but one that would distract from the main focus of this paper.

In an experimental contest pitting it against three auction formats that are standard in the theoretical and empirical literatures (the first price winner-pay, second price winner-pay and first price all-pay), we find that the bucket auction performs at least as well as theory predicts. On average, it raised three and a half times as much revenue as the better of the winner-pay mechanisms and one and a half times as much as the other all-pay mechanism.

In many cases the behavioral properties of agents complement the theoretical properties of mechanisms (e.g., Bowles and Polania-Reyes, 2012). With this in mind, our post-experiment survey was designed to gather data on behavioral predispositions that we conjectured might also explain behavior across the auction formats. Specifically, we conjectured that mechanisms in which participants submit sealed bids in the absence of information about the behavior of others might attenuate the “spirit of competition.” Second, we speculated that the tendency of real world bidders to be influenced by historical, as opposed to opportunity, costs (Kahneman and Tversky, 1984) could also affect revenue. Formats that ask for incremental bids, for example, might yield greater revenues from “sunk cost sensitive” bidders who tend to “throw good

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2 A raffle can be viewed as an inefficient all-pay auction in the sense that in equilibrium, the bidder who values the object most is only the most likely winner.

3 All-pay mechanisms may be less dominant in the field, however, because participation is both endogenous and a strong driver of revenues. In the field experiments described in Carpenter et al. (2008) the traditional first price winner-pay format did better than either the less common second price winner-pay and the largely unknown first price all-pay because people were less likely to participate under unfamiliar rules. This participation deficit in the all-pay has recently been replicated in the large-scale field study of Onderstal et al. (2013).
money after bad.\textsuperscript{4} Using the data from our survey, we show that some of the revenue difference accruing to the bucket auction can be attributed to these two behavioral factors. While the pure design effect of the mechanism is similar in magnitude to what theory predicts, the bucket auction raises considerably more money partially because it also entices sunk cost sensitive and competitive donors to raise their bids, both while not resulting in lower average welfare or diminished participation.

We proceed by first describing our experimental implementation of the bucket auction and the three comparison mechanisms and summarizing the demographic characteristics of our participants. We then provide theoretical foundations that allow us to form hypotheses about how well each auction mechanism should do for charity. Turning to the data, we first present summary measures of mechanism performance, control for other observables to show that our results are robust, and consider other factors that might affect a charity’s choice of mechanism. Because it is also important to show why the bucket does so well, we expand the analysis by demonstrating that the behavioral hypotheses that partially informed the design of our study (and the bucket format itself) account for much of the variation in revenue across mechanisms.

2. Implementing charity auctions in the lab

In this section, we highlight the unique and important features of our (between subjects) experimental design, one that allows the performance of the bucket auction to be compared with three other mechanisms. For comparison, we picked the three mechanisms that have received the most attention in both the theoretical and empirical literatures on charity auctions. Most importantly, we picked the three mechanisms for which there are theoretical predictions about expected revenue, conditional on endogenous participation, an important innovation of our design. For replication purposes and to be complete, the instructions for the bucket auction, which are representative of those used for all mechanisms, are included in Appendix A. Our complementary paper (Carpenter et al., 2010) was written to showcase the methodological details of the broader project: our randomization procedures, instructions for other (still unmodeled) mechanisms and our extensive post-experiment survey. The current paper is unique in that it provides theoretical foundations for the bucket auction, examines its relative revenue, efficiency, bidder welfare and fairness properties, and evaluates the extent to which several well-known behavioral tendencies also contribute to its success.

To this end, we conducted 20 auction sessions in which 197 participants were randomly assigned to one of four auction formats (five sessions per format): the first price winner-pay (FPWP) in which active participants submitted sealed bids, the high bidder won and paid her bid; the second price winner-pay (SPWP) where again sealed bids were collected, the high bidder won but only paid an amount equivalent to the second highest bid; the first price all-pay (FPAP), our third sealed bid auction, which differs from the FPWP only in that all bidders paid their bids, win or lose; and the bucket auction, as described earlier, successive bidders, arrayed randomly in a circle must either meet or exceed a fixed contribution on each round or leave the circle, with the prize awarded to the last bidder to contribute.

During each session, ten participants were faced with a sequence of ten auctions.\textsuperscript{5} The auctions were independent in the sense that earnings did not accumulate from round to round. To minimize the extent to which behavior in one round spilled over into the next, one round was chosen at random to determine final payoffs. On average, our participants earned $25.57 (with a low of $14 and a high of $31) for sessions that lasted approximately 1.5 hours and generated 200 revenue observations, 50 per format.

Drawing on the lessons of our field experiment (Carpenter et al., 2008), we implemented two design features intended to strengthen the external validity of our results. First, to minimize any “house money” effects which could cause bidders to be more willing to risk the monetary gains endowed by the experimenter (Thaler and Johnson, 1990), we had our participants earn the endowments later used to finance their bids. In the endowment stage of the experiment, participants had 12 minutes to solve as many anagrams as possible and were paid a piece rate of 10 experimental monetary units (EMUs) per correct word.\textsuperscript{6} Although we wanted the participants to feel as if they had earned their endowments, we also wanted to avoid possible income effects on bidding behavior so the word puzzles were chosen so that while it was relatively easy to get 10 out of 25 possible words correct, it was quite difficult to get more than 15 correct. Our manipulation seems to have worked mostly as planned: 73% of the participants correctly solved between the planned 10 and 15 puzzles and 94% solved between 10 and 16.\textsuperscript{7}

Second, we also wanted to allow participants to be able to “sit out” an auction, as potential bidders can and sometimes do at most real fundraisers. To this end, at the beginning of each of the ten auction rounds the participants were asked if they wanted to participate in the auction or do another anagram instead. To inform this decision, each participant was shown her endowment (which was private and replenished at the beginning of each round), her randomly assigned private value for the fictitious good at auction (which was drawn from $v_1 \in [0, 100]$ and changed from one round to the next), the

\textsuperscript{4} Ku et al. (2005), for example, show how “competitive arousal” or “auction fever” among participants can enhance revenues in sequential auctions. In a similar vein, the sequential nature of the bucket may also exploit the revenue enhancing properties of the “see and be seen” preferences observed in Salmon and Isaac (2006), Ku et al. (2005) also demonstrate how bidders are sensitive to sunk costs in sequential auctions.

\textsuperscript{5} Two sessions had fewer than ten participants. In one session nine attended and in the other eight did.

\textsuperscript{6} The exchange rate was 10 EMUs equaled a dollar.

\textsuperscript{7} The manipulation also worked in the sense that endowments do not differ by treatment and we do not find “income effects”: neither bids nor (in the aggregate) revenues correlate significantly with endowments.
prize for a correct answer to the puzzle (which was set to 15 EMUs on each round) and an indicator of how hard the puzzle would be. The puzzle difficulty changed randomly from one round to the next but each participant faced the same puzzle on any given round and the same sequence of puzzles was used for each session. After making the participation decision, participants were asked to estimate the number of people who would enter the auction on that round.

Endowments were subsequently used to finance bids. Because 98% of participants earned endowments of at least 100 EMUs, any bid up to the maximum value of 100 could be covered by one’s endowment. At the same time, bidders were not allowed to bid more than their endowments so bankruptcies were impossible. These choices seem to have been benign: in only 2% of instances does a person’s bid appear to be at all constrained by the endowment (i.e., bid = endowment).

To provide participants with an environment like that described in the theoretical literature (Goeree et al., 2005; Engers and McManus, 2006; Carpenter et al., 2009) in which private values are known, we induced values in a manner similar to Schram and Onderstal (2009). Specifically, for auction entrants, the direct benefit to winners was \( v_i - b_i \), the difference between private value and bid, in all but the SPWP, when it was equal to \( v_i - b_j \), where \( b_j \) was the second highest bid. The direct benefit to losers was either zero in the FPWP and SPWP or \( -b_j \) (that is, the forfeited bid) in the FPAP and bucket.8 Those people who opted out of the auction earned 15 EMUs if they solved the anagram and 0 EMUs otherwise. In addition, each participant in the session received indirect benefits equal to 10% of auction revenue, whether she bid on the item or not, and an additional “warm glow” (Andreoni, 1995) equal to 5% of any direct contribution. Under these rules, it is winners alone who experience a warm glow in the FPWP and SPWP, in contrast to the FPAP and bucket, in which all active bidders do.

The experiment was conducted anonymously (to mitigate any demand effects) and it was computerized using z-Tree (Fischbacher, 2007). At the beginning of each round endowments were replenished, values were randomly assigned and participants were shown their endowments, their current values and the difficulty of the current anagram. They were then asked whether they wanted to participate in the auction or do the puzzle. Those who chose to do the puzzle were presented a new anagram and had 2 minutes to solve it. Those who chose to enter any of the sealed bid formats were again shown their endowments and their values. These participants had 2 minutes to submit a bid. The computer randomly formed a circle from the active bucket bidders and picked one bidder to start. Bucket bidders were shown their endowments, their values, the current contents of the (virtual) bucket and the number of bidders left in the circle. When it was a bidder’s turn, she needed to contribute (at least) 5 EMUs to stay in the auction. The winner was the bidder who made the last contribution to the bucket.9 At the end of each round, participants, regardless of the format, were shown a summary of the auction, including the number of active bidders and the winning bid.

Table 1 summarizes the demographic characteristics of participants, disaggregated by mechanism.10 From an aggregate perspective, our efforts to recruit from a broader pool than usual were successful: the average participant was 25 years old but ages varied from 18 to 66. While 76% of our participants were college students, 19% were staff members and 5% were faculty. A little more than half (52%) of our participants were women, 76% were white, 24% had finished college or

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>FPWP</th>
<th>SPWP</th>
<th>FPAP</th>
<th>Bucket</th>
</tr>
</thead>
<tbody>
<tr>
<td>Participants</td>
<td>50</td>
<td>48</td>
<td>50</td>
<td>49</td>
</tr>
<tr>
<td>Age</td>
<td>25.94</td>
<td>23.42</td>
<td>25.26</td>
<td>27.16</td>
</tr>
<tr>
<td>Female (I)</td>
<td>0.58</td>
<td>0.46</td>
<td>0.48</td>
<td>0.57</td>
</tr>
<tr>
<td>White (I)</td>
<td>0.80</td>
<td>0.67</td>
<td>0.72</td>
<td>0.85</td>
</tr>
<tr>
<td>African American (I)</td>
<td>0.00</td>
<td>0.02</td>
<td>0.06</td>
<td>0.00</td>
</tr>
<tr>
<td>Asian (I)</td>
<td>0.16</td>
<td>0.19</td>
<td>0.14</td>
<td>0.08</td>
</tr>
<tr>
<td>Latino (I)</td>
<td>0.00</td>
<td>0.04</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>High School degree (I)</td>
<td>0.06</td>
<td>0.06</td>
<td>0.10</td>
<td>0.06</td>
</tr>
<tr>
<td>Some college (I)</td>
<td>0.70</td>
<td>0.75</td>
<td>0.70</td>
<td>0.61</td>
</tr>
<tr>
<td>College degree (I)</td>
<td>0.20</td>
<td>0.12</td>
<td>0.18</td>
<td>0.16</td>
</tr>
<tr>
<td>Graduate degree (I)</td>
<td>0.04</td>
<td>0.06</td>
<td>0.02</td>
<td>0.16</td>
</tr>
<tr>
<td>Income less than 25k (I)</td>
<td>0.16</td>
<td>0.18</td>
<td>0.12</td>
<td>0.06</td>
</tr>
<tr>
<td>Income between 25 and 50k (I)</td>
<td>0.12</td>
<td>0.12</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>Income between 50 and 75k (I)</td>
<td>0.14</td>
<td>0.23</td>
<td>0.26</td>
<td>0.10</td>
</tr>
<tr>
<td>Income between 75 and 100k (I)</td>
<td>0.22</td>
<td>0.19</td>
<td>0.06</td>
<td>0.24</td>
</tr>
<tr>
<td>Income between 100 and 125k (I)</td>
<td>0.18</td>
<td>0.12</td>
<td>0.12</td>
<td>0.16</td>
</tr>
<tr>
<td>Income between 125 and 150k (I)</td>
<td>0.06</td>
<td>0.04</td>
<td>0.12</td>
<td>0.04</td>
</tr>
<tr>
<td>Income more than 150k (I)</td>
<td>0.12</td>
<td>0.10</td>
<td>0.22</td>
<td>0.28</td>
</tr>
<tr>
<td>Charity auction experience (I)</td>
<td>0.24</td>
<td>0.21</td>
<td>0.16</td>
<td>0.28</td>
</tr>
<tr>
<td>No auction experience (I)</td>
<td>0.40</td>
<td>0.44</td>
<td>0.50</td>
<td>0.41</td>
</tr>
<tr>
<td>Endowment</td>
<td>142.40</td>
<td>144.17</td>
<td>143.80</td>
<td>144.49</td>
</tr>
</tbody>
</table>

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8 In the bucket, the bid is understood to equal the total amount contributed during the period.
9 There was no time limit on how long a bucket auction could last but they also tended to be done in about two minutes.
10 Few of the characteristics differ significantly by auction format indicating that our randomization procedure worked well.
received a higher degree, and 76% had annual family incomes greater than $50,000. In terms of experience, 44% had never participated in any form of auction before, but 22% had previously bid in a charity auction.

3. A theoretical framework

Engers and McManus (2006) provide a coherent framework for the analysis of the FPWP, SPWP and FPAP formats when the number of bidders is fixed. To review, consider an environment in which there are $N$ risk neutral bidders with independent private values ($v$) drawn from some common distribution $F$ over the unit interval. All bidders receive a benefit equal to a proportion $\alpha$ of auction revenues, and an additional warm glow equal to a fraction $\gamma$ of their own contribution. When, as is the case here, the distribution of private values is uniform, the symmetric equilibrium bid ($\sigma$) and implied expected revenue ($R$) functions can be shown to be:

$$
\sigma^{\text{FP}} = \left[ \frac{N-1}{(1-\gamma)N-\alpha} \right] v \\
\sigma^{\text{SP}} = \left[ \frac{1 + \left( \frac{1-\gamma}{\alpha} \right) v}{1 - \gamma} \right] \\
\sigma^{\text{AP}} = \left[ \frac{1}{1 - (\alpha + \gamma)} \right] \frac{N-1}{N} v^N \\
R^{\text{FP}} = \left[ \frac{N-1}{(1-\gamma)N-\alpha} \right] \left[ \frac{N}{N+1} \right] \\
R^{\text{SP}} = \frac{\alpha(N+1) + (1-\gamma)(N-1)}{(1-\gamma)(1+\alpha-\gamma)(N+1)} \\
R^{\text{AP}} = \frac{1}{1 - (\alpha + \gamma)} \left[ \frac{N-1}{N+1} \right]
$$

In the absence of both revenue proportional benefits ($\alpha = 0$) and warm glow ($\gamma = 0$), the three mechanisms are equivalent, with expected revenues equal to $\frac{3N}{4}$, a now familiar result. For the parameter values ($\alpha = 0.10$, $\gamma = 0.05$) induced in the lab, however, the pattern is quite different:

<table>
<thead>
<tr>
<th>$N$</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>...</th>
<th>10</th>
<th>25</th>
<th>50</th>
<th>100</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N-1$</td>
<td>0.33</td>
<td>0.50</td>
<td>0.60</td>
<td>0.67</td>
<td>...</td>
<td>0.82</td>
<td>0.92</td>
<td>0.96</td>
<td>0.98</td>
<td>1.00</td>
</tr>
<tr>
<td>$R^{\text{FP}}$</td>
<td>0.37</td>
<td>0.55</td>
<td>0.65</td>
<td>0.72</td>
<td>...</td>
<td>0.87</td>
<td>0.98</td>
<td>1.01</td>
<td>1.03</td>
<td>1.05</td>
</tr>
<tr>
<td>$R^{\text{SP}}$</td>
<td>0.42</td>
<td>0.58</td>
<td>0.67</td>
<td>0.74</td>
<td>...</td>
<td>0.88</td>
<td>0.98</td>
<td>1.02</td>
<td>1.03</td>
<td>1.05</td>
</tr>
<tr>
<td>$R^{\text{AP}}$</td>
<td>0.39</td>
<td>0.59</td>
<td>0.71</td>
<td>0.78</td>
<td>...</td>
<td>0.96</td>
<td>1.09</td>
<td>1.13</td>
<td>1.15</td>
<td>1.17</td>
</tr>
</tbody>
</table>

The principal feature of this data – that for $N \geq 3$, the all-pay mechanism produces more revenue, in expectation, than the second-price winner-pay, which in turn produces more than the first-price winner-pay, with little difference between the two winner-pay mechanisms in large auctions – is not specific to this example, and obtains for all admissible values of $\alpha$ and $\gamma$ of distributions of values.

To predict the relative performance of the bucket auction, we first note that it resembles a generalized war of attrition (Bulow and Klemperer, 1999) with positive externalities. The analogy isn’t perfect – the bucket auction’s discrete rounds, in which participants must decide, in some pre-determined order, whether or not to continue is not captured, for example – but it is close enough to be considered a useful approximation. We modify Bulow and Klemperer (1999) and (re)define the bid function $\sigma^B(v : \nu, k)$ to be the length of time a bidder with private value $v$ will remain in the auction when there are $k+1$ active bidders (or when $k$ bidders must drop out for the auction to end) and $\nu$ is the lowest possible valuation among still active bidders. If cost per unit time is normalized at 1, $\sigma^B(v : \nu, k)$ becomes the additional amount spent.

Two observations about this framework are warranted. First, if attention is limited to open-loop strategies – that is, if we suppose that bidders do not revise their strategies after others have dropped out or, in more formal terms, $\sigma^B$ is not allowed to vary with $\nu$ or $k$ – the bucket auction is then equivalent to a second-price all-pay (SPAP) mechanism. We know, for the reasons outlined in Coeere et al. (2005) and summarized earlier, that the SPAP format should produce more revenue than the FPAP, so there is (already) cause for optimism.

Second, if, however, bidders can revise their strategies, the auction has no symmetric equilibrium but, following Bulow and Klemperer (1999) once more, we consider a related auction, in which bidders must bear a continuation cost $c < 1$ per unit time after “leaving the circle” until the auction ends, and then focus on the limit case $c \to 0$.

Such costs do not matter, of course, when $k = 1$, and the auction ends with the next exit. We relegate the derivation, which echoes Bulow and Klemperer (1999), to Appendix B, but it is not difficult to show that the optimal bid function is:

$$
\sigma^B(v : \nu, 1) = \frac{1}{1 - 2\alpha - \gamma} \int_0^v h(x)dx
$$

where $h(x)$ is the hazard rate of the value distribution. In the uniform [0, 1] case, this becomes, after some simplification:

$$
\sigma^B(v : \nu, 1) = \frac{1}{1 - 2\alpha - \gamma} \left( -(v - \nu) + \ln \frac{1 - \nu}{1 - v} \right)
$$
It is worth noting the (implicit) effects of the previous $N - 2$ exits on the behavior of the last two participants: as auction size $N$ increases, so, too, will the threshold $v$ and with it, the time they’re prepared to remain in the circle. Both bidders understand that when there are more participants, the expected value of the other’s valuation is higher.

In the case $k > 1$, when more than two bidders remain, a modification of the Bulow and Klemperer (1999) argument, which we outline in Appendix B, implies that in the uniform case:

$$
\sigma_B^k (v : v, k) = \frac{c^{k-1}}{1 - 2\alpha - \gamma} \left( -(v - v) + \ln \frac{1 - v}{1 - v} \right)
$$

which is the unique symmetric perfect Bayesian equilibrium, characterized by monotonic bid functions.

As continuation costs approach 0, and the model more closely resembles a bucket auction, we are left with the sharp and testable prediction that all but the two strongest or highest value bidders will drop out (almost) immediately. The “survivors” then employ the subgame strategy $\sigma_B^k (v : v, 1)$, for example, when three bidders start the auction, the one with the lowest value of the three drops out immediately and the remaining two calculate bids based on $v = v_3$.

It then follows, by a simple extension of Bulow and Klemperer’s (1999) first corollary, that the expected length of the bucket auction will be $\frac{1}{2(1 - 2\alpha - \gamma)}E (v_2)$ where $E (v_2)$ is the expected value of the second highest value, or $N - 1$ when the distribution of values is uniform. Expected revenues are twice this or, in this case, $\frac{1}{2(1 - 2\alpha - \gamma)} (\frac{N - 1}{N + 1})^{11}$.

To characterize its relative performance, consider Fig. 1, which plots, as a function of the number of bidders $N$, the expected revenue gains of the bucket, FPAP and SPWP, all relative to the FPWP which serves as the benchmark for our empirical work. The size of the “bucket premium” is remarkable: with $N = 10$ bidders, for example, it exceeds 25%, more than the 10% “all-pay premium” and much more than the vanishing “second-price premium,” which is smaller than 1%. Furthermore, not only does the bucket premium increase with the number of bidders, it increases more, in absolute terms, than other premia.

### 4. Mechanism performance

As the previous section details, we expect the bucket auction to do well compared to other standard benchmarks. Our evaluation of the bucket starts with participation, both because the number of bidders affects revenue but also because, under some circumstances, event planners who seek to cultivate a community of donors will want to involve more people whenever it’s reasonable to do so. Because the opportunity cost of participating (the expected payoff from trying another anagram) is exogenous and all players, regardless of participation, accrue revenue proportional benefits, our model’s stark

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11 A referee has asked whether our focus on the limiting case $c \to 0$ is a reasonable approximation since participants could not leave the experiment after “dropping out of the circle.” The question is an important one because bids, or length of time in the circle, are monotonic in continuation costs, which implies that our theoretical estimate of the “bucket premium” is too small. A careful examination of the data, however, suggests that the approximation is, in this context, a reasonable one. The variant of the Bulow and Klemperer (1999) model described here implies that when continuation costs are bounded above zero, the expected time between exits increases, which, as Fig. 3, confirms, is not what we observe. Furthermore, the data also suggest that once behavioral influences have been accounted for, the size of the bucket premium is more or less consistent with this approximation. For further discussion of both observations, see the discussion in Sections 4 and 6.
predictions assume that the number of active bidders is either fixed or, if endogenous, then independent of mechanism. A natural first question, therefore, is whether this is indeed the case and, if not, what the revenue implications might be. We see, at the top of Table 2, that participation rates hover between 40 and 50 percent. Consistent with the field experiment reported in Carpenter et al. (2008), fewer people took part in the FPAP, but the bucket enticed as many people or more to participate than any other mechanism.\textsuperscript{12} It follows, therefore, that low participation is not a characteristic feature of every all-pay format.

Fundraisers are also interested in the likelihood of auction failure, which occurs when no one enters or, in the SPWP, when the second highest bid is zero. Clearly, when there are no bidders, failure may just be inconvenient (i.e., another event must be organized); however, our implementation was strict in that when participants did enter and the second highest bid was zero, the item was forfeited. This strict interpretation of the rules can impose a substantial cost and so it would benefit sellers to add a clause that the second highest price must be positive. Considering the data, we report failure rates on the last line of Table 2, and are not surprised that the SPWP fails more often (7/50 auctions) than the others but also find that the failure rate is close to zero for the other mechanisms.

The second line in Table 2 summarizes our main results. On average, the SPWP yields only 67.90 EMUs, which is slightly less than the other winner-pay mechanism, the FPWP, which yields 75.59 EMUs. This difference could be attributed to the difference in failure rates: if one excludes failed auctions, mean revenues are 78.95 and 78.74, respectively. Notice these revenue levels are remarkably close to those predicted at the beginning of Section 3. As seen in previous studies (discussed in Section 1), we also find that the FPAP does considerably better than either winner-pay format, with mean revenues equal to 178.33 EMUs.\textsuperscript{13} However, the bucket, which produces an average of 264.18 EMUs, dwarfs even this. Its revenues are more than three times either winner-pay format and one and a half times that of the other all-pay. Fig. 2 provides a visual representation of these results, including the 95% confidence intervals around revenue means, and reinforces our basic result: the FPAP does significantly better than either winner-pay while the bucket does significantly better than any other format.\textsuperscript{14}

We cultivate a better sense of these differences in Table 3 which reports estimates for three revenue models. Because revenues are truncated at zero in the auctions that fail despite attracting a positive number of bidders, we use a Tobit spec-

\begin{table}[h]
\centering
\caption{Mean auction characteristics by mechanism ($N = 200$).}
\begin{tabular}{|l|c|c|c|c|}
\hline
 & FPWP & SPWP & FPAP & Bucket \\
\hline
Participation rate & 0.462 & 0.465 & 0.424 & 0.490 \\
Revenue & 75.59 & 67.90 & 178.33 & 264.18 \\
Failed auctions & 0.04 & 0.14 & 0.00 & 0.02 \\
\hline
\end{tabular}
\end{table}

\textsuperscript{12} More specifically, while the differences in participation between the bucket and either the FPWP or the SPWP are not quite significant, the bucket does draw significantly more people than the FPAP ($p = 0.05$).

\textsuperscript{13} Other related studies also find that the FPAP does considerably better than both the FPWP (e.g., Schram and Onderstal, 2009) and theory (e.g., Gneezy and Smorodinsky, 2006). While Lugovskyy et al. (2010) have shown that all-pay bids fall somewhat when the experiment is repeated a large number of times and when the partners matching protocol is used, even these changes do not eliminated overbidding.

\textsuperscript{14} Using the nonparametric Wilcoxon test, the $p$-values are less than 0.01 for each relevant comparison.
Table 3  
Testing for revenue differences.

<table>
<thead>
<tr>
<th></th>
<th>(1) No controls</th>
<th>(2) No controls</th>
<th>(3) Controls</th>
</tr>
</thead>
<tbody>
<tr>
<td>Second-price winner-pay (I)</td>
<td>−18.296***</td>
<td>−12.137***</td>
<td>−18.254***</td>
</tr>
<tr>
<td></td>
<td>(5.715)</td>
<td>(10.200)</td>
<td>(14.065)</td>
</tr>
<tr>
<td>First-price all-pay (I)</td>
<td>99.596***</td>
<td>111.506***</td>
<td>122.981***</td>
</tr>
<tr>
<td></td>
<td>(10.050)</td>
<td>(11.038)</td>
<td>(8.890)</td>
</tr>
<tr>
<td>Bucket (I)</td>
<td>190.832***</td>
<td>190.243***</td>
<td>198.463***</td>
</tr>
<tr>
<td></td>
<td>(22.587)</td>
<td>(22.078)</td>
<td>(19.089)</td>
</tr>
<tr>
<td>Number of active bidders</td>
<td>20.803***</td>
<td>20.964***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(4.485)</td>
<td>(4.188)</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>197</td>
<td>197</td>
<td>197</td>
</tr>
</tbody>
</table>

Controls include session size, age, gender, ethnicity, education, income and previous auction experience.

Tobit estimates of revenue; (robust standard errors) clustered on session.

* p < 0.10.
** p < 0.05.
*** p < 0.01.

Fig. 3. Bucket auctions converge to wars of attrition between two bidders.

To be sure that it is not just participation differences that drive the divergence in revenue, the second column of Table 3 adds the number of active bidders to the model. Despite being the “thickest” auction, on average, it does not appear that the bucket's performance is the result of participation differences. As expected, more bidders increase revenue but controlling for participation does little to diminish the estimated revenue differences. The last column of Table 3 shows that when additional auction-level controls for the number of participants in the session, the average age of the participants, and the fractions of the participants who are women, white, who have completed at least some college, who have high family incomes and who have some auction experience are included, the mechanism effects are not much affected. In short, as predicted the strength of the bucket is a robust feature of the data.

To offer one last, quite different, perspective on our results, we reinforce the similarities between bucket auctions and wars of attrition in Fig. 3, which plots the mean number of active bucket bidders in each observed cycle – that is, a round

15 The three auctions in which nobody participated are dropped from the sample, as they are in Table 4 which analyzes the effects of the different predispositions of active bidders. However, the results of Table 3 are very similar when least squares is used on the full sample. In addition, the results are virtually identical if instead of clustering the errors, we include period fixed effects or session random effects.

16 The revenue differences are also not a consequence of differences in how difficult, or not, the rules are: we asked participants about this after each session, and found no significant differences across mechanisms.
through the circuit of active bidders. As one can see the longest bucket auction lasted 32 cycles (and earned 380 EMUs), although auctions lasting even 25 cycles were quite common. However, the interesting question motivated by our model (and Bulow and Klemperer, 1999) is just how quickly these auctions distill to just two active bidders. As the figure suggests, although many start the auction, bidders quickly leave the bucket in the first five cycles. Indeed, the decay in participation is roughly exponential: after getting quickly to three, attrition slows a bit but after the 15th cycle the strong mode is for just two bidders to be left.

5. **Other performance metrics**

It is reasonable to wonder, however, whether the revenue prowess of the bucket comes at some other cost. Does the bucket perform relatively poorly on other metrics of auction success? As mentioned above, charities also have an incentive to not spoil their donor base by using an auction format that seems unfair. Likewise, auctions in which bidder welfare is low (perhaps due to overbidding in a war of attrition) or there is regret because the prize tends to end up in the wrong hands might also erode the donor base. We examine each of these potential criticisms (fairness perceptions, sustained participation, bidder welfare and efficiency) in turn.

In our post-experimental questionnaire we asked our participants to rate the “fairness” (the term was purposefully broad) of the session format on a 5-point Likert scale. Despite performing poorly, the SPWP was ranked the fairest, significantly so, most likely because naive bidders expected winners to pay less. At the same time, the bucket was ranked no less fair than the FPWP and significantly more fair than the FPAP, perhaps because bucket bidders appreciated the ability to titrate their bids in response to information about the number of active bidders – a feature unavailable in seal bid formats.

Concerning participation, we already know from Table 2 that overall participation is high in the bucket, but what we do not know is whether enthusiasm is high initially, perhaps because of the novelty of the format, and then quickly falls off as participants learn in later rounds to avoid charitable wars of attrition. As Fig. 4, which plots the mean number of entering bidders by round suggests, there is no evidence that the bucket, or for that matter any other mechanism, fizzles with repeated exposure. Participation does seem to cycle a bit but, as the overlaid pins in Fig. 4 reveal, this is just a consequence of the random variation in puzzle difficulty.17 In more precise terms, even those who contest long wars of attrition return to participate in subsequent auctions. Whether or not this feature proves robust in the field is an important question for future research, but it does provide at least casual evidence of an additional winner’s (or even competitor’s) benefit.18

As an indicator of bidder welfare, exclusive of these additional benefits, we present box plots of the surpluses, inclusive of both private and revenue proportional returns, achieved in each of the formats in Fig. 5. It is clear that the range of surplus values is larger for the bucket than the other formats and although overbidding (i.e., bidding more than one’s subsidized value) clearly occurs in every format (in fact, it is almost as endemic in the FPAP), nearly a quarter of the outcomes are negative for the bucket, a fact that is consistent with approximately one quarter (i.e., 2 or 3) of the participants typically remaining in the auction each round. While this may seem like a black mark for the bucket, one must remember the context.

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17 This extends to revenues as well. Adding either time period indicators (as discussed above) or the puzzle difficulty to the last column of Table 3 also has little effect on the mechanism point estimates.

18 As one referee observes, it will also be important to measure “auction length” in the field. While there were no memorable time differences in the lab, and few sessions ran longer than the scheduled one hour, the emergence of differences in the field would have welfare implications.
In charity auctions there are large positive externalities to bidding wars of attrition. In fact, as the white lines indicate, the median surplus is actually highest in the bucket because all the other bidders received large revenue proportional benefits. To the extent that those with the largest private values also tend to be those with the highest incomes, the mechanism has an important redistributive feature.

The remaining concern is efficiency – in what fraction of the auctions does the prize end up in the hands of the person with the highest (induced) value? This fraction is plotted for each mechanism in Fig. 6. Across mechanisms, 58% of auctions are efficient and there seems to be a broad difference between winner-pay and all-pay formats with the winner-pay formats being approximately 10 percentage points more efficient, on average. The bucket, however, is not the least efficient auction on average and, given the confidence intervals, there are actually no statistically significant differences among the mechanisms.

In the end, not only does the bucket auction raise considerably more revenue than the other formats, it also does not appear to disappoint potential donors. Participation remains high even with repeated exposure, the large externalities generated by the bidding wars keep most bidders happy, and it is no less efficient that the other, more standard, mechanisms.

6. Behavioral factors

Is it the case that the strategic environment created by the bucket, as outlined in Section 3, is enough to generate such stark revenue differences or are there other, perhaps behavioral, factors that also contribute to the bucket’s revenue premium?
As we alluded in the introduction, we hypothesized from the start that bidders who were sunk cost sensitive might remain in the bucket auction longer, and that competitive bidders would be more prone to engage in bidding wars in the bucket. With these hypotheses in mind, we added questions to our post-experiment survey to measure both the sunk cost sensitivity and competitiveness of our participants.

To assess sunk cost sensitivity, participants were asked variants of two standard questions (Thaler, 1980; Kahneman and Tversky, 1984):

1. Imagine that you have decided to see a movie in town and have purchased a $10 ticket. As you’re waiting outside the theatre for a friend to join you, you discover that you’ve lost the ticket. The seats are not marked and the ticket cannot be recovered because the person who sold it doesn’t remember you. Would you buy another $10 ticket?
2. Imagine that a month ago, you and a friend made a nonrefundable $100 deposit on a hotel room in Montreal for the coming weekend. Since the reservation was made, however, the two of you have been invited to spend the same weekend at another friend’s cottage in Vermont. You’d both prefer to spend the weekend at the cottage but if you don’t go to Montreal, the $100 deposit will be lost. Would you still go to Montreal?

Participants whose answers to both were indicative of sunk cost sensitivity were labeled as such, and 21% of our sample met this criterion.

We asked two more direct questions to evaluate competitiveness. The first was purposefully broad (and in the spirit of Ryckman et al., 1990), while the second was motivated by a sense that at least some respondents would find it more socially acceptable to characterize themselves as competitive in sports (à la Gill and Deeter, 1988):

1. In general, how competitive do you think that you are?
2. Concerning just sports and leisure activities, how competitive do you think that you are?

Respondents used a 10 point Likert scale to answer both questions, and were classified competitive if the sum of their scores was 15 or more. Using this cutoff, 41% of our participants were competitive, but it is important to note that the results are robust with respect to other plausible thresholds.19

As a first step we examine whether sunk cost sensitivity and competitiveness translate into remaining in the bucket auction longer. Fig. 7 illustrates survival functions that compare those participants classified as sunk cost sensitive to those who were not (panel a) and those characterized as competitive or not (panel b). The graphs also control for the influence of the other behavioral bias. As one might expect in wars of attrition, the general survival trend suggests that fewer than half the participants who enter the bucket make it 10 cycles around the circle (spending 50 EMUs) but more importantly, and confirming our prior, both sunk cost sensitive and competitive bidders remain in the bucket longer. In fact, both log-rank tests for the pairwise comparisons and a Cox proportional hazard model suggest that the behavior of sunk cost sensitive and competitive bidders is significantly different: controlling for all the other demographic effects, the hazard ratio of sunk cost sensitive bidders is 0.59 and it is 0.65 for competitive bidders (with p < 0.01 for both).

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19 One might also worry that because the survey came after the experiment, one’s experience might shape one’s response to these questions. Although not definitive, it is reassuring that the behavioral measures do not vary significantly between treatments.
Table 4
Testing behavioral hypotheses.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Baseline</td>
<td>Sunk costs?</td>
<td>Competitiveness?</td>
<td>Both?</td>
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<tr>
<td>Second-price winner-pay (I)</td>
<td>−11.816</td>
<td>3.786</td>
<td>−31.310*</td>
<td>−32.585**</td>
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<tr>
<td></td>
<td>(11.174)</td>
<td>(17.308)</td>
<td>(17.688)</td>
<td>(16.380)</td>
</tr>
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<td>First-price all-pay (I)</td>
<td>119.513***</td>
<td>129.711***</td>
<td>75.432***</td>
<td>75.092***</td>
</tr>
<tr>
<td></td>
<td>(10.526)</td>
<td>(15.491)</td>
<td>(22.267)</td>
<td>(17.410)</td>
</tr>
<tr>
<td>Bucket (I)</td>
<td>206.521***</td>
<td>179.043***</td>
<td>69.377*</td>
<td>48.475</td>
</tr>
<tr>
<td></td>
<td>(14.763)</td>
<td>(20.247)</td>
<td>(37.149)</td>
<td>(37.729)</td>
</tr>
<tr>
<td>Sunk cost sensitive bidders (N)</td>
<td>−8.551</td>
<td>−3.914</td>
<td>−3.035</td>
<td>−0.325</td>
</tr>
<tr>
<td></td>
<td>(7.111)</td>
<td>(7.546)</td>
<td>(5.173)</td>
<td>(10.320)</td>
</tr>
<tr>
<td>Sunk cost sensitive bidders × SPWP</td>
<td>−13.593</td>
<td>−8.904</td>
<td>−34.943***</td>
<td>−13.593</td>
</tr>
<tr>
<td></td>
<td>(12.248)</td>
<td>(11.211)</td>
<td>(10.589)</td>
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<td>Sunk cost sensitive bidders × FPAP</td>
<td>40.813**</td>
<td>−18.622***</td>
<td>35.927***</td>
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<td></td>
<td>(13.758)</td>
<td>(8.934)</td>
<td>(9.786)</td>
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<td>Competitive bidders (N)</td>
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<td>−8.914</td>
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<td></td>
<td>(6.994)</td>
<td>(7.546)</td>
<td>(7.546)</td>
<td>(7.546)</td>
</tr>
<tr>
<td>Competitive bidders × SPWP</td>
<td>16.714**</td>
<td>18.622***</td>
<td>35.927***</td>
<td>35.927***</td>
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<tr>
<td></td>
<td>(6.954)</td>
<td>(6.610)</td>
<td>(9.786)</td>
<td>(9.786)</td>
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<tr>
<td>Competitive bidders × FPAP</td>
<td>23.329***</td>
<td>35.927***</td>
<td>35.927***</td>
<td>35.927***</td>
</tr>
<tr>
<td></td>
<td>(8.934)</td>
<td>(9.786)</td>
<td>(9.786)</td>
<td>(9.786)</td>
</tr>
<tr>
<td>Competitive bidders × bucket</td>
<td>69.055***</td>
<td>68.020***</td>
<td>68.020***</td>
<td>68.020***</td>
</tr>
<tr>
<td></td>
<td>(20.626)</td>
<td>(20.845)</td>
<td>(20.845)</td>
<td>(20.845)</td>
</tr>
</tbody>
</table>

Controls include session size, age, gender, ethnicity, education, income and previous auction experience. Tobit estimates of revenue; (robust standard errors) clustered on session.

* p < 0.10.
** p < 0.05.
*** p < 0.01.

In Table 4, we use mediation analysis to assess the extent to which these behavioral factors can explain the revenue success of the bucket auction. The first column largely (because of the included controls) replicates Table 3 and serves as our baseline specification. Again, we see that the revenue ordering is bucket > FPAP > FPWP > SPWP. In column (2) we add the number of sunk cost sensitive participants in the session and its interactions with each of the formats. This specification exhibits two important features. First, while the interaction of the number of sunk cost sensitive bidders and the bucket indicator is both large and highly significant, none of the other interactions are, suggesting that bucket auctions better exploit the presence of such bidders. Second, the point estimate of the bucket coefficient falls by 13% relative to the baseline, which implies that some of the variation in revenue previously attributed to the bucket mechanism itself is in fact the differential responsiveness of auction formats to the sunk cost sensitive bidders.
In column (3) of Table 4 we analyze the effect of competitiveness and find that it is an even stronger mediating factor. As was the case with sunk cost sensitivity, the interaction of the number of competitive bidders and the bucket indicator is both large and statistically significant. The interactions with the SPWP and FPAP are also statistically significant but much smaller, suggesting that competitive bidders are more much aggressive in bucket auctions than other formats. Crucially, inclusion of the number of competitive bidders in an auction (and its interactions) reduces the estimated bucket coefficient by almost 66%, from 206.52 to 69.37, and this estimate is statistically significant at just the 10% level. The inference we draw is that the chance for bidders to compete in bucket auctions is a principal reason it does so well.

To complete the analysis, column (4) incorporates both behavioral hypotheses. The most salient consequence is that the bucket coefficient shrinks even further, and is no longer statistically significant at the 10% level. At the same time, it is important to note that taking the coefficient at face value (i.e., the effect of the bucket in auctions like those modeled in Section 3 with no sunk cost sensitive or competitive bidders), the estimate of a 48 EMU premium constitutes approximately a 60% (48/76) increase over the FPWP which, while much smaller than the estimates in Table 3, is in the ballpark of the 25–30% increase predicted by theory (i.e., Fig. 1). In short, only a fraction of the bucket auction’s success can be ascribed to its theoretical properties – it appears that the behavioral factors matter a lot. Also, while the addition of one more sunk cost sensitive bidder is estimated to increase auction revenues a little more than 30 EMUs in a bucket auction relative to the FPWP benchmark, the addition of one more competitive bidder is estimated to increase revenue by almost 70 EMUs.

7. Concluding remarks

Given the predictable failure of revenue equivalence in charity auctions, there is a lot of freedom for economists to think creatively about auction mechanisms that can enhance fundraising. It is our view that the way forward must build on both theoretical and behavioral insights. In particular, our focus on all-pay mechanisms is rooted in current models of charity auctions but our conjecture that such mechanisms could do even better if redesigned to exploit the competitiveness of some bidders and/or their penchant to emphasize historical, as opposed to opportunity, costs finds its rationale in experimental and other data. We believe that the success of the bucket auction in the lab, a simple mechanism that embodies these considerations, validates our approach.

It remains to be seen, however, whether the bucket auction will pass the ultimate test: is it as effective at real fundraisers? While the issue of scalability will also have to be explored, field pilots that we have conducted make us curious to know what might happen if we could bring a bucket to the next Robin Hood Foundation auction, especially given it is often attended by so many (undoubtedly competitive) hedge fund managers.

Acknowledgments

We thank Phil Mellizo, Stella Nordhagen and Wesley Pech for research assistance, John Spraggon and John Stranlund for organizing access to the experimental lab at the University of Massachusetts and Olivier Bos, Carolyn Craven, Marco Faravelli, Steve Holmes, Mark Isaac, Corinna Noelke, Sander Onderstal, Tim Salmon and the seminar participants at St. Andrews University, McGill University, George Mason University, the University of Massachusetts – Amherst, and the Canadian Economics Association conference.

Appendix A. Bucket auction instructions

Introduction

Today you are participating in a decision making experiment. You will earn $10 just for showing up. The instructions are straightforward, and if you follow them you may be able to make a considerable amount of money. During the experiment, all decisions will be framed in terms of ‘experimental monetary units,’ or EMUs. At the conclusion of the experiment, all the EMUs that you have accumulated will be converted into real dollars at the rate of 10 EMUs per real dollar (i.e., we will divide your EMUs by 10). You will be paid in cash at the end of the experiment.

Please read these instructions carefully, as understanding the rules is essential for doing well. You may refer to these instructions at any time during the experiment. If you have any questions while these instructions are being read, please raise your hand and we will attempt to answer them.

The experiment consists of three phases, all conducted using the computer: in the first, you will earn an amount of money, your ‘endowment,’ in the second you will be able to use those earnings to take part in an auction, if you so choose, and in the last phase you will complete a brief survey. Your final payoff will depend upon your performance in the first phase and your own actions as well as the actions of the other participants in the second phase.

Experiment phase one: endowment

During the first part of the experiment, the ‘endowment phase,’ you will be asked to solve a series of word scrambles – puzzles in which the letters of a word are mixed up. It is your task to unscramble them. On your computer screen you will see one scrambled word at a time, with a blank below each given letter. In each blank, enter the letter that you think belongs in that space in the correct, unscrambled word.
You will have a total of 12 minutes to correctly solve as many scrambles as you can, and for each that you solve correctly, you will earn an additional 10 EMUs. The puzzles increase in difficulty as you progress, and you will have only one chance to solve each puzzle. You may leave a puzzle blank, but once you click the ‘Submit and Continue to Next Puzzle’ button, you will not be able to return to that puzzle. There are a total of 25 scrambles. You will not know how many you have solved correctly until the phase is over.

Once you have reached the end of the puzzles, please sit quietly and wait for other participants to finish. At the end of the phase, the number of puzzles you solved correctly and the total EMUs you earned will be shown to you. This amount of EMUs constitutes your endowment and will be used to participate in the second phase of the experiment.

**Experiment phase two: auction**

**Motivation**

In the second phase of the experiment we simulate a charity auction. Charity auctions are different from regular, for profit, auctions because everyone associated with the charity benefits from the money that is raised. In non-charity auctions, only winners benefit. To simulate this difference, participants in these charity auction simulations will earn benefits from three sources: they earn benefits from winning the auction, they earn benefits from the total amount of money raised by the auction, and they earn benefits from their own contributions. The second source of benefits represents the fact that everyone benefits when money is contributed to charity and the third source represents the fact that people often feel good about themselves for giving money.

**Deciding whether to participate and bidding**

In the second phase of the experiment, there will be ten periods. At the beginning of each period you will decide whether you want to participate in an auction or try to solve another word scramble. In the auction, you will have the opportunity to bid on a single unit of a fictitious good. Although the good is fictitious, it will have some real ‘value’ to you – you can think of this as being the amount of money that the experimenter would pay you for the item if you obtained it in the auction. Each participant will learn his or her value for the item at the beginning of each period, but will not know any of the other participants’ values. Other participants will have different values. Your value for the good will change each period, and how this value is determined is described in detail below.

If you choose to participate in the auction, you will submit bids for the fictitious commodity. This will be done by adding money to a ‘bucket’ which holds all the bids. Each participant will be able to bid by paying money, at least 5 EMUs at a time, into the bucket. The bucket will be passed from one participant to another in an order which is randomly set at the beginning of the period. Once you have placed money in the bucket, it cannot be taken back and must be paid out of your endowment, so you cannot put more than your endowment into the bucket.

At any time during the auction when it is your turn to add money to the bucket, you can ‘pass’ which means you pass the bucket to the next participant without adding anything. Once you pass you will be removed from the bidding for the period. The auction will end when there are two people left in the auction and one passes. The winner of the auction will be the person who last put money into the bucket, even if that person has not added the most in total. Bids and values will both be denominated in EMUs. How auction gains are determined is described in the next section.

As indicated above, participation in the auction is a choice. Before you decide to enter a bid or solve a scramble you will be shown the value you will have for the fictitious good in the auction and the difficulty of the scramble you will have to solve. If you choose not to participate in the auction, you will have 2 minutes to solve the word scramble. If you solve it within the time limit, you will earn 15 EMUs; if you do not, you will earn 0 EMUs. The difficulty of the puzzle will change randomly at the beginning of each period but the difficulty is the same for all scramble solvers within a period.

**Auction rules and determining profits**

After all but one of the bidders have passed, the auction will end and the last person to add to the bucket will win. The revenue generated by the auction is the total amount in the bucket – the amount paid by all participants. As mentioned above, this revenue has value for all participants, regardless of whether they participate in the auction or try the scramble: each person earns 0.10 times the total auction revenue – the second source of benefits referred to above. The amount each bidder contributes to the bucket has an additional value for them, so that each bidder earns an additional 0.05 times the amount (s)he placed in the bucket. This is the third source of benefits mentioned above.

We can work through an example to illustrate the payoffs. Suppose that 6 people have entered the auction – let’s call them Arthur, Barbara, Charles, Diane, Ethan and Frances – and that four others have attempted the scramble. Suppose, too, that after Diane added some EMUs to the bucket, Ethan, Francis, Arthur, Barbara and Charles all passed, so that Diane has won the auction. To calculate how much Diane gains or loses from this win, we need to know how much she values the object, how much she contributed to the bucket and how much all of the others contributed. Suppose, for the sake of argument, that the object is worth 50 EMUs to her, she contributed 10, and that the five other bidders put a total of 90 EMUs in the bucket.

Diane’s direct gain, the difference between what the object is worth and what she paid for it, is 50 – 10 or 40. Because 100 EMUs have been contributed to the bucket – the 10 that Diane contributed and the 90 that Arthur, Barbara, Charles, Ethan and Frances combined contributed – each of them, and each of the four non-bidders, will receive an additional benefit worth 0.10 × 100 or 10 EMUs because the charity raised 100 EMUs of revenue from the auction.

And last but not least, Diane’s good feeling is worth 0.05 of her contribution or 0.05 × 10 = 0.5 EMUs to her.
Altogether, Diane’s direct and additional gains are therefore equal to $40 + 10 + 0.5 = 50.5$ EMUs. If she started the auction with an endowment of, say, 120 EMUs, she would leave it with $120 + 50.5 = 170.5$ EMUs.

What about someone like Arthur, who didn’t win the auction? Let’s suppose that Arthur’s endowment was 110 EMUs, that the object was worth 40 EMUs to him and that he added 30 EMUs to the bucket. (Even if Arthur contributes more than Diane, he does not win the auction if he is not the last one to add to the bucket.) Arthur’s direct gain is $-30$ EMUs (that is, he suffers a direct loss) since he bids 30 EMUs but does not win the object. The first of his two additional gains is the same as Diane’s, or $0.10 \times 100 = 10$ EMUs, while the second is $0.05 \times 30 = 1.5$ EMUs because he put 30 EMUs in the bucket.

Altogether, Arthur’s net gain is $-30 + 10 + 1.5$ or $-18.5$ EMUs. Since he entered the auction with an endowment of 110 EMUs, he leaves with 91.5 EMUs.

Finally, what about those who attempted the word scrambles? Let’s consider the hypothetical cases of Gerry, who does not solve his scramble, and Hannah, who does solve her scramble.

Gerry doesn’t earn the 15 EMUs for solving the scramble but he does receive the $0.10 \times 100 = 10$ EMUs that each of the bidders and non-bidders received in this auction. If he started with the auction with an endowment of 120 EMUs, he therefore ends it with 130 EMUs.

Hannah earns 15 EMUs for her scramble and the $0.10 \times 100 = 10$ EMUs that all participants receive so her combined gain is 25 EMUs. If she started the auction with an endowment of 130 EMUs, for example, she ends it with 155 EMUs.

In algebraic terms, the earnings of any participant can be summarized as follows:

$$\text{Winning Bidder Earnings} = \left[\text{Endowment} + (\text{Value} - \text{Amount Placed in Bucket})\right] + 0.10 \times (\text{Total Amount in Bucket})$$

$$+ 0.05 \times \text{(Amount Placed in Bucket)}$$

Where $\text{Amount Placed in Bucket}$ is the amount added by just this bidder, and $\text{Total Amount in Bucket}$ is the amount added by all bidders combined. The total earnings for an auction participant who is not the last one to add to the bucket are:

$$\text{Earnings of other Bidders} = (\text{Endowment} - \text{Amount Placed in Bucket}) + 0.10 \times (\text{Total Amount in Bucket})$$

$$+ 0.05 \times \text{(Amount Placed in Bucket)}$$

The earnings of people who choose to try the scramble instead of participating in the auction are, if you get it right:

$$\text{Earnings of Scrambler} = (\text{Endowment} + 15) + 0.10 \times (\text{Total Amount in Bucket})$$

if you get it wrong:

$$\text{Earnings of Scrambler} = \text{Endowment} + 0.10 \times (\text{Total Amount in Bucket})$$

**Determination of final dollar payoffs**

After 10 rounds of the auction have been played, the computer will randomly pick one round to count towards your final earnings from the experiment. Because the computer will pick one round randomly, each auction period is completely independent of the others (i.e., you do not accumulate gains or losses from one period to the next). However, if you make losses in the auction phase they will be deducted from the money you earn in the first, endowment phase. The computer will report to you the randomly chosen round and your final payoff in EMUs. After the questionnaire stage is complete, the computer will report your earnings in dollars. All data collected in the experiment will be anonymous and used only for academic research. You will be paid privately, and no other participant will be told what you earned in the experiment.

**Auction details**

*How are the values generated?*

Values are chosen randomly from the interval 0 to 100 EMUs. Your value is independent of the values of all other experiment participants and of your value from other rounds: knowing your value in a given round tells you nothing about the values of other experiment participants, and knowing your values in previous rounds tells you nothing about your value in the current round. All values between 0 and 100 are equally likely.

*How much and how little can I add to the bucket?*

You can add as little as 5 EMU, and as much as you like as long as the total amount you have placed in the bucket does not exceed your endowment. The bucket will start out empty. If all active bidders pass the bucket before anyone adds money to it, the auction ends, there is no winner and auction revenues and all auction participants’ gains are zero. If this is the randomly chosen round for payment, all the auction participants will just earn their phase one endowments.
Appendix B. Derivation of a "bucket" equilibrium

First, observe that when \( k = 1 \), and the auction ends with the next exit, the marginal cost to a bidder with value \( v \) of remaining in the auction a little longer, enough to outlast a rival with value between \( v \) and \( v + dv \), will be \((1 - (\alpha + \gamma'))\sigma_B(v ; y, 1)dv\), where \( \sigma_B = dv/dv \) denotes the derivative of the bid function with respect to value \( v \). That is, she will spend \( \sigma_B(v ; y, 1)dv \) additional units of time or dollars, but because she will also earn a common return \( \alpha \) and warm glow \( \gamma \) for each dollar contributed, the net cost per unit time is \( 1 - (\alpha + \gamma') \).

The decision to delay exit is associated with two distinct marginal benefits. First, with likelihood \( h(v)dv \), where \( h(v) = f(v) \) is the standard hazard rate, her rival will now be the first to exit, and she will receive the prize worth \( v \) to her. Second, win or lose, her decision to remain in the auction forces her rival to remain in a little longer than she would otherwise have, too, and so produces an additional revenue proportional benefit for all bidders equal to \( \alpha\sigma_B(v ; y, 1)dv \).

A necessary condition for the optimality of \( \sigma_B(v ; y, 1) \) is therefore \((1 - (\alpha + \gamma'))\sigma_B(v ; y, 1) = h(v) + \alpha\sigma_B(v ; y, 1) \) or, after the collection of terms:

\[
(1 - 2\alpha - \gamma')\sigma_B(v ; y, 1) = h(v) \nu
\]

The solution is:

\[
\sigma_B(v ; y, 1) = \frac{1}{1 - 2\alpha - \gamma'} \left( -(v - \nu) + \ln \frac{1 - \nu}{1 - \nu} \right)
\]

where the second equality follows from the observation that a bidder with the lowest possible valuation \( \nu \) should immediately drop out, in which case \( \sigma_B(v ; y, 1) = 0 \). As an aside, the solution also presupposes stricter limits on the common return and warm glow than our comparison formats: unless \( 2\alpha + \gamma' < 1 \), marginal benefits will always exceed marginal costs. In the uniform \([0, 1]\) case, the bid function is, after some simplification:

\[
\sigma_B(v ; y, 1) = \frac{1}{1 - 2\alpha - \gamma'} \left( -(v - \nu) + \ln \frac{1 - \nu}{1 - \nu} \right)
\]

which is the result in the text. It is worth noting the (implicit) effects of the previous \( N - 2 \) exits on the behavior of the last two participants: as auction size \( N \) increases, so, too, will the threshold \( \nu \) and with it, the time they’re prepared to remain in the circle. Both bidders understand that when there are more participants, the expected value of the other’s valuation is higher.

In the case \( k > 1 \), when more than two bidders remain, we extend the intuition in Bulow and Klemperer (1999) and observe that, to a first order approximation, incremental delay no longer increases the probability of winning. Nor does it increase the length of the auction and, therefore, the revenue proportional benefits that accrue from others’ bids. Because it reduces the time a bidder bears the smaller costs of continuation \( c \), however, she would not be indifferent between delay and exit unless others delayed, too, such that \( \sigma_B(v ; y, k) = \alpha\sigma_B(v ; y, k - 1) \), so that \( \sigma_B(v ; y, k) = c^{k-1}\sigma_B(v ; y, 1) = h(v) \nu \). Using the previous result, this implies, in the uniform case:

\[
\sigma_B(v ; y, k) = \frac{c^{k-1}}{1 - 2\alpha - \gamma'} \left( -(v - \nu) + \ln \frac{1 - \nu}{1 - \nu} \right)
\]

which is the result in the text.

References


