Instructions:

- You must attempt Problem 1.
- Please attempt at least three of Problems 2,3,4,5.
- If you attempt all five problems then your final score will be the sum of your score for Problem 1 and the scores for the three remaining problems receiving the highest number points.
- Calculators are not permitted.

1. (10 points) True/False:
   (a) \[
   \lim_{{(x,y) \to (0,0)}} \frac{x^2 y}{{x^2 + y^3}} = 1
   \]
   (b) If the partial derivatives of \( f(x) \) exist at \( a \) then \( f \) is differentiable at \( a \).
   (c) If \( \frac{\partial f}{\partial x}(P) = 0 \) then \( f(x) \leq f(P) \) for all \( x \) in a small disc centred at \( P \).
   (d) Let \( f(x, y) \) be function such that \( f(tx, ty) = t^3 f(x, y) \), for all \( t \). Then,
      \[
      x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = 3f(x, y)
      \]
   (e) The vector field \( F = \begin{bmatrix} x^2 \sin(xy) \\ y^2 \cos(xy) \end{bmatrix} \) is conservative.

2. Consider the conservative vector field on \( \mathbb{R}^2 \)
   \[
   F = \begin{bmatrix} 3x^2 y + y^3 + 1 \\ x^3 + 3xy^2 + 2 \end{bmatrix}
   \]
   (a) Determine the potential function \( f(x, y) \) for \( F \) satisfying \( f(-1,0) = 0 \).
   (b) Determine the tangent line to the level curve \( f(x, y) = 0 \) at \((-1,0)\).
   (c) Show that the tangent line to the level curve \( f(x, y) = 3 \) at \((0,1)\) is parallel to the tangent line computed in (b).

3. Consider the function
   \[
   f : \mathbb{R}^2 \to \mathbb{R}, \quad (x, y) \mapsto x^2 y^2 - x - y
   \]
   (a) Determine \( \nabla f(3,2) \)
   (b) Determine the equation of the tangent plane to the graph of \( f(x, y) \) at \((3,2,3)\).
   (c) Use a linear approximation to find the approximate value of \( f(2.9,2.1) \).
(d) Compute the directional derivative of $f$ at $(3,2)$ in the direction $v = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$.

4. Consider the differentiable functions

$$f : \{(x, y) \mid x > 0\} \to \mathbb{R}^2, \begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} y/x \\ x^2 + y^2 \end{bmatrix}$$

$$g : \mathbb{R}^2 \to \mathbb{R}^2, \begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} 2xy \\ x^2 - y^2 \end{bmatrix}$$

$$h : \mathbb{R}^2 \to \mathbb{R}, (x, y) \mapsto x^3 + 3xy + y^3$$

(a) Compute the Jacobian matrices $Df$, $Dg$, $Dh$.

(b) Let $p(x, y) = g(y/x, x^2 + y^2)$. Determine $Dp$.

(c) Let $q(x, y) = h(2xy, x^2 - y^2)$. Determine $\nabla q$ and compute $\frac{\partial q}{\partial y}(1,1)$.

5. A function $f(x, y)$ has the following level curve diagram

(a) On the level curve diagram mark the portion(s) of the level curve $f = 2$ where $\frac{\partial f}{\partial y} \geq 0$. 

![Level Curve Diagram]
(b) Based on the information given in the level curve diagram, mark the correct relationship between $|\nabla f(A)|$ and $|\nabla f(B)|$:

| $|\nabla f(A)| < |\nabla f(B)|$ | $|\nabla f(A)| = |\nabla f(B)|$ | $|\nabla f(A)| > |\nabla f(B)|$ |
|-----------------|-----------------|-----------------|
|                 |                 |                 |

(c) Let $\underline{v} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$. On the level curve diagram mark a point $C$ satisfying the following conditions:

$$\frac{\partial f}{\partial x}(C) < 0, \quad \text{and} \quad D_{\underline{v}}f(C) > 0$$