Linear Algebra
Exam 1
Spring 2007

March 15, 2007

Name:
Honor Code Statement:

Directions: Complete all problems. Justify all answers/solutions. Calculators are not permitted.

1. **Define:** Given vectors $\mathbf{v_1}, \mathbf{v_2}, \ldots \mathbf{v_p}$ in $\mathbb{R}^n$ then the **subset of** $\mathbb{R}^n$ **spanned by** $\mathbf{v_1}, \mathbf{v_2}, \ldots \mathbf{v_p}$ is ...

2. **Define** what it means for a mapping to be **onto**. Give an example of such a mapping.

3. Assuming that T is a linear transformation, find the standard matrix of T, where $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a vertical shear transformation that maps $\mathbf{e_1}$ into $\mathbf{e_1} - \mathbf{2e_2}$, but leaves the vector $\mathbf{e_2}$ unchanged.

4. Is the following matrix singular? Why, or why not?

$$A = \begin{bmatrix} 1 & 6 \\ -1/2 & 2 \end{bmatrix}$$

Compute the inverse of this matrix and use it to solve the equation $A\mathbf{x} = \mathbf{b}$, where

$$b = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

5. Is the following set of vectors linearly dependent? If not, give a justification. If so, give a linear dependence relation for them.

$$v_1 = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

$$v_2 = \begin{bmatrix} -4 \\ -5 \\ -6 \end{bmatrix}$$

$$v_3 = \begin{bmatrix} 6 \\ 3 \\ 0 \end{bmatrix}$$

6. **True or False:** Justify each answer by citing an appropriate definition or theorem. If the statement is false and you can provide a counterexample to demonstrate this then do so. If the statement is false and be can slightly modified so as to make it true then indicate how this may be done.

   - If the columns of an $n \times n$ matrix A are linearly independent, then the columns of A span $\mathbb{R}^n$.

   - There exists a one-to-one linear transformation mapping $\mathbb{R}^3$ to $\mathbb{R}^2$.

   - If one row in an echelon form of an augmented matrix is $[0\ 0\ 0\ 5\ 0]$, the the associated linear system is inconsistent.

   - An inconsistent linear system has more than one solution.

   - The codomain of the transformation $\mathbf{x} \mapsto A\mathbf{x}$ is the set of all linear combinations of the columns of A.

7. Determine if the following matrices are invertible. Use as few calculations as necessary. Justify your answer.

$$A_1 = \begin{bmatrix} 1 & 0 & 7 & 5 \\ 0 & 1 & 98 & 3 \\ 0 & 0 & 33 & -9 \\ 0 & 0 & 0 & 11 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 1 & 0 & -2 & 11 \\ 2 & 1 & -4 & 11 \\ 3 & 0 & -6 & 11 \\ 4 & 0 & -8 & 11 \end{bmatrix}$$

8. Compute the product $A_2 A_1$ (using the matrices from the previously problem).

Show in as few calculations possible that this product does not equal $A_1 A_2$.

9. Given $\mathbf{v_1}, \mathbf{v_2} \neq \mathbf{0}$ and $\mathbf{p}$ in $\mathbb{R}^3$. Further, $\mathbf{v_1}$ is not a scalar multiple of $\mathbf{v_2}$.

   - Give a geometric description of the parametric equation $\mathbf{x} = \mathbf{p} + s\mathbf{v_1} + t\mathbf{v_2}$.

   - Given a linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$. Describe the image of the set of vectors satisfying the above parametric equation under T.

10. Given a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$. Prove the following statement: If T is one-to-one then the equation $T(\mathbf{x}) = \mathbf{0}$ has only the trivial solution. Do NOT claim this is true by the Invertible Matrix Theorem. (Note that the IMT would only apply if $n = m$.)